Two-sided Auctions with Budgets: Fairness, Incentives and Efficiency

Extended Abstract

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ABSTRACT

This paper considers the fairness in the problem of budget-feasible mechanism design in two-sided markets where multiple sellers come with indivisible items and buyers come with budgets. Buyers could untruthfully claim their budgets to procure as much value of items as possible from sellers. Each seller with a single item is required to bid his cost since the cost is privately known, while the value of each item is publicly known. A viable mechanism should satisfy buyers' fairness where a buyer with more budget can procure more value of items, and budget feasibility where buyers' respective budgets are not exceeded. The goal is to investigate budget-feasible mechanisms that guarantee the fairness, incentives and efficiency simultaneously. We consider two models by distinguishing the types of items, one with homogeneous items and one with heterogeneous items. Our main contributions are the budget-feasible mechanisms for these models that guarantee the fairness, the truthfulness both on the sellers' side and the buyers' side, and constant approximation to the optimal total procured value from sellers.

KEYWORDS

Two-sided auctions; Mechanism design; Fairness; Budget feasibility

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1 INTRODUCTION

Being one of the important factors usually addressed in the economic markets, *fairness* illustrates an individual's judgment or evaluation for the appropriateness and rationality of a process or an action. Besides, it shows each entity's acceptance or satisfaction of its outcome, for example, related to a decision or a result [1, 6–8]. For two-sided markets, mechanism design plays an important role in dealing with many real-world markets, e.g., [3, 5]. One more essential issue in two-sided markets is to design budget-feasible mechanisms where payments from buyers should not exceed their respective budgets and was initially studied in the single-buyer setting [2, 4, 10] and then extended to two-sided markets with multiple buyers [11]. When multiple buyers have different budgets, they would have diverse abilities to procure items from sellers. In order to improve the satisfaction and willingness of participants, there is a natural requirement to consider the *fairness* with respect to budgets that the buyer with more budget should procure more values of items from sellers than the buyers with less budget in the two-sided markets. However, few prior works have considered similar concept of fairness with respect to budgets or procurement abilities in two-sided auctions. Therefore, we attempt to design approximate budget-feasible mechanisms in two-sided markets that guarantees the fairness, incentives and efficiency simultaneously.

2 PRELIMINARIES

Two-sided Market Model: There are n *sellers* $S = \{s_1, ..., s_n\}$ and m *buyers* $A = \{a_1, ..., a_m\}$ in the two-sided procurement market. Each buyer a_i has a private *budget* $B_i \in R_+$. Each seller s_j has an item with common known *value* $v_j \in R_+$ and private *cost* $c_j \in R_+$ to sell. Let $B = \{B_1, ..., B_m\}$ denote all the budgets of the buyers. Let $C = \{c_1, ..., c_n\}$ be all the costs of the sellers and $V = \{v_1, ..., v_n\}$ be all the values of the items. Denote by B_{-i} and C_{-j} all budgets except a_i's budget B_i and all costs except s_j's cost c_j, respectively. Following the assumption in [11], we assume that all buyers have basic procurement ability $B_i \geq B_{min}$ where B_{min} is publicly known minimum threshold of budget and no items of sellers exceed any buyer's procurement ability, i.e., $c_j \leq B_{min}$.

We focus on the strategic scenario where the participants (buyers and sellers) may act strategically to maximize their own utilities. Each seller bids its cost b_j of its item. Let $b = \{b_1, b_2, ..., b_n\}$ denote all the bids of the sellers. Each buyer a_i claims a budget B_i' that may be different from its true budget B_i. Following the assumption in [11], we assume that each buyer is required to submit the full amount of his claimed budget as deposit to the mechanism at the beginning, so that a buyer bidding over its true budget would be detected and punished with infinite cost.

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The Mechanism: Formally, a mechanism $M = (f, P)$ consists of an allocation function f and a payment function P. In our work, we consider indivisible item model. We use $x_{ij} = \{0, 1\}$ to indicate whether the item of seller s_j is allocated/sold to buyer a_i, and define x_j as $x_j = \sum_{1 \le i \le m} x_{ij}$. Similarly we use p_{ij} to denote how much payment is paid from a_i to s_j. We say sellers' item values are homogeneous if their values are the same or heterogeneous otherwise. The utility of seller s_j is the difference between the payment it receives and its true cost, i.e., $u_j(b_j) = \sum_{1 \le i \le m} p_{ij} - c_j x_j$. The utility of buyer a_i is the total value of items that are bought from sellers within its budget, $u_i(B_i) = \sum_{1 \le j \le n} v_j x_{ij}$. Our objective is to maximize the total value of the buyers procured from the market, following the assumption in the classical reverse-auctions [9, 10], denoted by $V(S, B) = \sum_{1 \le i \le n} \sum_{1 \le j \le m} v_j x_{ij}$.

The proposed mechanisms need to guarantee the following properties, **1) Individual rationality.** The utility of a winning seller s_j is non-negative, i.e., $u_j(b_j) \ge 0$. **2) Computational efficiency.** The output of the mechanism should be computed in polynomial time. **3) Sellers'/buyers' truthfulness.** Any seller maximizes its own utility when its bid b_j equals its true cost c_j, i.e., $u_j(c_j, C_{-j}) \ge u_j(b_j, C_{-j}), \forall b_j \ne c_j$. Any buyer maximizes its own utility when its claimed budget equals its true budget, i.e., $u_i(B_i, B_{-i}) \ge u_i(B'_i, B_{-i}), \forall B'_i \ne B_i$. **4) Budget balance (BB).** *Strong budget-balance (SBB)* means that the amount of money paid by the buyers is totally and exclusively transferred to the sellers. It is *weak budget-balance (WBB)* if the mechanism does not run a deficit. **5) Budget feasibility.** The total payment of each buyer a_i does not exceed its budget, i.e., $\sum_{1 \le j \le n} p_{ij} \le B_i$. **6) Fairness.** We want to guarantee the fairness between buyers in the sense that a buyer with more budget can procure more values from sellers. That is, we say a mechanism satisfies *weak fairness* if $u_{i_1}(B_{i_1}) \ge u_{i_2}(B_{i_2})$ when $B_{i_1} \ge B_{i_2}$. If $u_i(B_i) = \frac{B_i}{\beta}$, we say the mechanism satisfies *strong β-fairness*. **7) Approximation.** Ideally, we would like our mechanism to be $O(1)$-approximation that the ratio between the optimal solution and the solution by the mechanism is $O(1)$.

3 MECHANISM DESIGN

Mechanism in homogeneous item model: We first design Mechanism HomoMech (HM for short) to solve the situation where the items of sellers are homogeneous. The high level idea of the HM is as follows. To guarantee buyers' fairness, we propose an idea of *virtual (unit) price* and use it to measure the demand and the supply of participants in the market. Based on the generated demand curve and supply curves as the virtual unit price q changes, we will match the demand with the supply in a proper manner to guarantee the fairness, truthfulness and efficiency simultaneously. Specifically, we will introduce a set of *candidate virtual prices*, denoted as Q. We use $N_b(q)$ to denote all buyers' supportable number of items when the virtual price is set with value $q \in Q$, i.e., $N_b(q) = \sum_{1 \le i \le m} \lfloor \frac{B_i}{q} \rfloor$, and use $N_s(q)$ to denote the total number of candidate sellers who bid cost no more than q, i.e., $N_s(q) = |\{b_j | b_j \le q, b_j \in b\}|$. Note that as the virtual price q decreases continuously, $N_b(q)$ is non-decreasing while $N_s(q)$ is non-increasing, hence we can find the points that either $N_b(q)$ or $N_s(q)$ changes. Among these prices, we try to find a *critical virtual price* q^* and correspondingly another value $\tilde{q}$ to instruct us to determine the allocation and the payment of our mechanism, which is able to further elicit truthfulness from the sellers and buyers simultaneously. Figure 1 gives an illustration of the demand and supply curves.

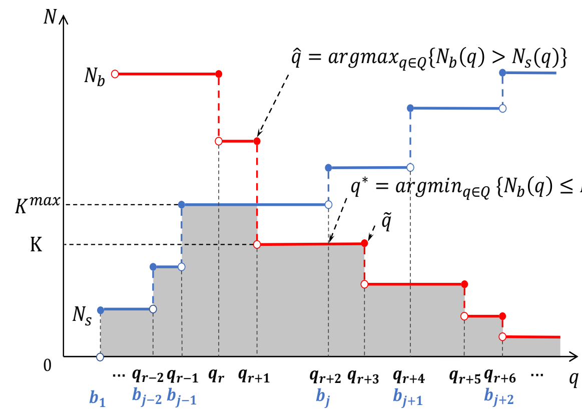

Figure 1: Demand and supply curves (solid lines) as the virtual price q changes. The red step function ($N_b(q)$) measures how many items buyers could buy at price q, while the blue one ($N_s(q)$) measures how many items sellers could sell at q. The dark area indicates the number of procurable items at different q. While the maximum procurable number is K^{max}, the mechanism decides to procure K items.

For theoretical performance, we prove that mechanism HM satisfies the budget-feasibility, individual-rationality, computational efficiency, strong budget-balance, sellers'/buyers' truthfulness, weak fairness and achieves an approximation ratio of $2 + \frac{m-1}{K} \le 3$.

Mechanism in heterogeneous item model: We further design a randomized Mechanism MHIM to solve the case where the items of the sellers have heterogeneous values. MHIM randomly combines two sub-mechanisms as follows. We divide the sellers into two groups. To tackle the sellers with *small bids* (no greater than $\frac{B_{min}}{3}$), we design a sub-mechanism PM to elicit the fairness. The main idea of PM is to partition the sellers into virtual unit-value sellers and call the truthful and efficient Mechanism HM to output a virtual allocation, and then generate a real allocation to make each buyer maximize its expected utility in a random manner. For sellers with *large bids* (greater than $\frac{B_{min}}{3}$), we apply a simple sub-mechanism SM. Then, MHIM combines PM and SM randomly.

For theoretical performance, we prove that MHIM guarantees budget feasibility, individual rationality, computational efficiency, strong budget balance, sellers'/buyers' truthfulness, weak fairness, and achieves 12-approximation.

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