

# Sequential Voting in Multi-agent Soft Constraint Aggregation\*

JAAMAS Track

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## ABSTRACT

We study a sequential preference aggregation procedure based on voting rules for settings where several agents express their preferences over a common set of variable assignments via soft constraints. We evaluate this approach by providing both theoretical and experimental results.

## KEYWORDS

Preferences in multi-agent systems; Sequential voting; Soft constraint aggregation.

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## 1 INTRODUCTION

We consider scenarios where a set of agents needs to select a common decision from a set of possible decisions, over which they express their preferences. When the set of decisions is small, agents may just present an order over them to express their preferences. However, when the set of objects is very large, as often in real-life situations, this is unfeasible. This occurs in several AI applications, such as combinatorial auctions, web recommender systems, and configuration systems [17]. In this paper we assume that the decision set has a combinatorial structure. Fortunately, in the presence of such a combinatorial structure (i.e. candidates are described by feature vectors), agents may describe their preference in a compact and efficient way, using one of the several formalisms available in the literature, such as soft constraints [16], CP-nets [3], and graphical utility models [2]. Often, we must make a joint decision and we need to compromise our preferences with those of other people: in this work we assume agents to compactly express their preferences over the candidates via soft constraints, a compact way to model

preferences which naturally models variables domains, and relationship among variables. The goal is to aggregate such preferences and to select a joint decision via voting rules [1] (as alternative to other approaches such as [7], etc.). In our context, where the number of candidates is very large, it may take too much time to provide a voting rule with the preference orderings of the agents over such candidates. A valid alternative to this is to consider a sequential approach (computationally more attractive) that uses a voting rule several times, on each feature of the decision set. Thus to aggregate the preferences of the agents, we consider a sequential procedure that asks the agents to vote on one variable at a time. We study several classical properties of this procedure, by relating them to corresponding properties of the adopted voting rules used for each variable. Moreover, we perform an experimental study on a special kind of soft constraints, namely fuzzy constraints. A similar approach has been considered for CP-nets in [9] as well as [22], [20], while for variants of CP-nets have been considered in [4, 8, 10]. The sequential procedure with soft constraints presented in this paper has also been studied in terms of its resistance to bribery in [6, 11–13, 15, 18, 19].

## 2 BACKGROUND

**Soft constraints** A soft constraint [21] involves a set of variables and associates a preference value from a (totally or partially ordered) set to each instantiation of its variables. Such a value is taken from a preference structure  $S = \langle A, +, \times, 0, 1 \rangle$ , where  $A$  is the set of preference values,  $+$  induces an ordering over  $A$  (where  $a \leq b$  iff  $a + b = b$ ),  $\times$  is used to combine preference values, and 0 and 1 are respectively the worst and best element. A Soft Constraint Satisfaction Problem (SCSP) is a tuple  $\langle V, D, C, A \rangle$  where  $V$  is a set of variables,  $D$  is the domain of the variables,  $C$  is a set of soft constraints (each one involving a subset of  $V$ ) associating values from  $A$ . An instance of the SCSP framework is obtained by choosing a specific preference structure. Choosing  $S_{FCSP} = \langle [0, 1], \max, \min, 0, 1 \rangle$  means that preferences are in  $[0, 1]$  and we want to maximize the minimum preference. This is the setting of fuzzy CSPs (FCSPs).

An optimal solution of an SCSP is a complete assignment with an undominated preference. Finding an optimal solution is an NP-hard problem, unless certain restrictions are imposed, such as a tree-shaped constraint graph. Constraint propagation may help the search for an optimal solution. For the purposes of this paper, it is enough to consider a specific form of constraint propagation

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called directional arc consistency (DAC). DAC is enough to find the preference level of an optimal solution when the constraint graph of the problem has no cycles (and thus it has a tree shape), since the optimum preference level is the best preference level in the domain of the root variable [21].

**Voting rules** A voting rule [1] allows a set of voters to choose one among a set of candidates. Voters need to submit their vote, that is, their preference ordering over the set of candidates (or part of it), and the voting rule aggregates such votes to yield a result, usually called the winner. Given a profile (a collection of total orderings over the set of candidates), a *voting rule* maps it onto a single winning candidate. Some examples of widely used voting rules, that we will use in what follows, are (we assume a tie-breaking mechanism to assure a single winner): *Plurality*, *Borda*, *Approval*, *Copeland*. The properties of voting systems are desirable also in automated contexts. We will use a few of them (for details see [1]), since we will later be interested in studying their presence (or absence) in the preference aggregation system we propose: *Condorcet-consistency*, *Anonymity*, *Neutrality*, *Monotonicity*, *Consistency*, *Participation*, *Efficiency*, *Independence of Irrelevant Alternatives*, *Non-dictatorship* and *Strategy-proofness*.

### 3 THE METHOD

The idea is to sequentially vote on each variable via a voting rule, possibly using a different voting rule for each variable (similarly to the approach for CP-nets in [9]). Thus, our approach uses a voting rule several times, on each feature of the decision set following a specific order. That is, the voting rule asks the agents to provide their preferences on each feature at a time, and at each step a winner value for a certain feature will be returned. At the end, the collection of winner values will constitute the winning candidate.

A *soft profile* is a triple  $(V, D, P)$  where  $V$  is a set of variables (also called issues),  $D$  is a sequence of  $|V|$  ordered finite domains, and  $P$  a sequence of  $m$  SCSPs over variables in  $V$  with domains in  $D^1$ . A *fuzzy profile* is a soft profile  $(V, D, P)$  where  $P$  is a sequence of  $m$  fuzzy CSPs. In this paper, we consider soft profiles where each voter expresses his/her preferences via an SCSP with a tree-shaped constraint graph.

Considering a soft profile  $(V, D, P)$  with  $|V| = n$ , an ordering of such variables  $O = \langle V_1, \dots, V_n \rangle$ , and a corresponding sequence of voting rules  $R = \langle r_1, \dots, r_n \rangle$  (that will be “local”), the *sequential procedure* we propose is a sequence of  $n$  steps, where each step  $i$  corresponds to<sup>2</sup>: **1)** All agents are asked for their preference ordering over the values in the domain of variable  $V_i$ , yielding profile  $p_i$  over such domain (performing DAC on their SCSP, following  $O$ ). **2)** The voting rule  $r_i$  is applied to  $p_i$ , returning a winning assignment  $d_i$  for  $V_i$ . **3)** The unary constraint  $\langle f_i, \{V_i\} \rangle$  on the variable  $V_i$  is added to the SCSPs of each agent, where  $f_i$  associates the preference value 1 to  $d_i$  and 0 to all the values in the domain of  $V_i$  different from  $d_i$ . **4)** If the new SCSPs is not be DAC, DAC algorithm is applied following  $O$ . After all  $n$  steps have been executed, the winning assignments ( $Seq_{O,R}(V, D, P)$ ) are collected in the tuple  $\langle d_1, \dots, d_n \rangle$ , i.e., the winner of the election.

<sup>1</sup>Notice that a soft profile consists of a collection of SCSPs over the same set of variables, while a profile (as in the classical social choice setting) is a collection of total orderings over a set of candidates.

<sup>2</sup>All the ties are broken lexicographically if needed.

	Local. $\rightarrow$ Seq.	Seq. $\rightarrow$ Local
Condorcet Consist.	No	Yes
Efficiency	Yes (unique top)	Yes
Anonymity	Yes	Yes
Neutrality	No	Yes
Consistency	Yes	Yes
Participation	No	Yes
Monotonicity	Yes	Yes
IIA	No	Yes
Non-dictatorship	Yes	Yes
Strategy-proofness	No	Yes

**Table 1: Property preservation.**

Rule	ADO nonSeq.	ADO Seq.	$\Delta(\text{ADO})$ nonSeq. – Seq.	$\Delta(\text{Time})$ nonSeq. – Seq.
<i>Plurality</i>	0.4220	0.4369	-0.0149	-0.0087
<i>Approval</i>	0.3846	0.3829	0.0017	-0.0091
<i>Borda</i>	0.3974	0.4307	-0.0333	16.9771
<i>Copeland</i>	0.4092	0.4619	-0.0527	834.8084

**Table 2: Rules comparison: ADO and computational time.**

This sequential approach is more attractive computationally, since usually the number of values of each feature is small. However, when features are interdependent, it is not clear if the result of this sequential approach is useful at all. In this paper we consider this issue, assuming agents express their preferences via soft constraints.

### 4 RESULTS

**Theoretical results** We consider a soft profile  $(V, D, P)$  where each voter expresses his/her preferences via an SCSP with a tree-shaped constraint graph. If the sequential voting procedure satisfies a given property, so do all the local voting rules. The opposite holds for anonymity, consistency, efficiency, monotonicity, and non-dictatorship. These results are summarized in Table 1. In particular, we consider a sequential voting procedure where at each step we apply the local voting rule  $r_i$  to variable  $X_i$ , that is,  $Seq_{\langle X_1, \dots, X_n \rangle, \langle r_1, \dots, r_n \rangle}$ . The second column describes results regarding whether a property satisfied by all  $r_i$  is also satisfied by  $Seq_{\langle X_1, \dots, X_n \rangle, \langle r_1, \dots, r_n \rangle}$ , while the third column does the opposite. Notice that one of the results regarding efficiency holds only in the restricted case occurring when all the ordering induced by the SCSPs have a single top element.

**Experimental results.** To compare the four considered voting rules, we analyse their preference and ADO (i.e., the average distance of the winner outcome from the optimal outcome) on randomly generated profiles with tree-shaped FCSPs with 25 voters, 5 issues, 5 domain elements per issue, and 20% tightness. We also consider, how they vary from a non sequential approach to a sequential approach looking at their ADO and computation time. We show the results in Table 2.

In our synthetic profiles each agent’s FCSP is generated randomly. Thus the probability that two voters vote equally is very small and this implies a large amount of disagreement among the agents. We also consider more realistic profiles, with data from TripAdvisor (from PrefLib [14]), with very similar results as shown above.

## REFERENCES

- [1] K. J. Arrow, A. K. Sen, and K. Suzumura. 2002. *Handbook of Social Choice and Welfare*. North-Holland.
- [2] F. Bacchus and A.J. Grove. 1995. Graphical models for preference and utility. In *Proceedings of UAI 1995*. 3–10.
- [3] C. Boutilier, R. I. Brafman, C. Domshlak, H. H. Hoos, and D. Poole. 2004. CP-nets: A Tool for Representing and Reasoning with Conditional Ceteris Paribus Preference Statements. *JAIR* 21 (2004), 135–191.
- [4] V. Conitzer and L. Xia. 2012. Paradoxes of Multiple Elections: An Approximation Approach. In *Proceedings of KR 2012*.
- [5] C. Cornelio, M.P. Pini, F. Rossi, and K.B. Venable. 2019. Multi-agent soft constraint aggregation via sequential voting: theoretical and experimental results. *Autonomous Agents and Multi-Agent Systems* 33, 1 (2019), 159–191. <https://doi.org/10.1007/s10458-018-09400-y>
- [6] P. Faliszewski, E. Hemaspaandra, and L. A. Hemaspaandra. 2009. How Hard Is Bribery in Elections? *JAIR* 35 (2009), 485–532.
- [7] C. Gonzales, P. Perny, and S. Queiroz. 2008. Preference Aggregation with Graphical Utility Models. In *Proceedings of AAI 2008*. 1037–1042.
- [8] J. Lang, J. Mengin, and L. Xia. 2012. Aggregating Conditionally Lexicographic Preferences on Multi-issue Domains. In *Proceedings of CP 2012*. 973–987.
- [9] J. Lang and L. Xia. 2009. Sequential composition of voting rules in multi-issue domains. *Mathematical social sciences* 57 (2009), 304–324.
- [10] X. Liu and M. Truszczynski. 2013. Aggregating Conditionally Lexicographic Preferences Using Answer Set Programming Solvers. In *Proceedings of International Conference on Algorithmic Decision Theory (ADT-13)*.
- [11] A. Maran, N. Maudet, M. S. Pini, F. Rossi, and K. B. Venable. 2013. A Framework for Aggregating Influenced CP-Nets and its Resistance to Bribery. In *Proceedings of AAAI 2013*.
- [12] N. Mattei, M. S. Pini, K. B. Venable, and F. Rossi. 2012. Bribery in voting over combinatorial domains is easy. In *Proceedings of AAMAS 2012*. 1407–1408.
- [13] N. Mattei, M. S. Pini, K. B. Venable, and F. Rossi. 2013. Bribery in voting with CP-nets. *Annals of Mathematics and Artificial Intelligence* (2013).
- [14] Nicholas Mattei and Toby Walsh. 2013. PrefLib: A Library of Preference Data. In *Proceedings of ADT 2013*. Springer.
- [15] N. Maudet, M. S. Pini, K. B. Venable, and F. Rossi. 2012. Influence and aggregation of preferences over combinatorial domains. In *Proceedings of AAMAS 2012*. 1313–1314.
- [16] P. Meseguer, F. Rossi, and T. Schiex. 2005. Soft constraints. In *Handbook of Constraint Programming*, F. Rossi, P. Van Beek, and T. Walsh (Eds.). Elsevier.
- [17] S. Mittal and F. Frayman. 1989. Toward a generic model of configuration tasks. In *Proceedings of IJCAI 1989*. 13951401.
- [18] M. S. Pini, F. Rossi, and K. B. Venable. 2013. Bribery in Voting With Soft Constraints. In *Proceedings of AAAI 2013*.
- [19] M. S. Pini, F. Rossi, and K. B. Venable. 2013. Resistance to bribery when aggregating soft constraints. In *Proceedings of AAMAS 2013*. 1301–1302.
- [20] K. Purrington and E. H. Durfee. 2007. Making social choices from individuals' CP-nets. In *Proceedings of AAMAS 2007*. 1122–1124.
- [21] F. Rossi, P. Van Beek, and T. Walsh. 2006. *Handbook of Constraint Programming*. Elsevier.
- [22] L. Xia, V. Conitzer, and J. Lang. 2010. Aggregating Preferences in Multi-Issue Domains by Using Maximum Likelihood Estimators. In *Proceedings of AAMAS 2010*. 399–408.