A Logic of Inferable in Multi-Agent Systems with Budget and Costs

Extended Abstract

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ABSTRACT

In AI, Multi-Agent Systems are able to model many kind of collective behavior and have therefore a wide range of application. In this paper, we propose a logical framework (Logic of "Inferable") which enable reasoning about whether a group of agents can perform an action, highlighting the concepts of *cost* of actions and of *budget* that agents have available to perform actions. The focus is on modeling the group dynamics of cooperative agents.

KEYWORDS

Multi Agents Systems; Modal Logic; Epistemic Logic

ACM Reference Format:
Stefania Costantini, Andrea Formisano, and Valentina Pitoni. 2021. A Logic of Inferable in Multi-Agent Systems with Budget and Costs: Extended Abstract. In *Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021), Online, May 3–7, 2021*, IFAAMAS, 3 pages.

1 INTRODUCTION

Multi-agent systems are widely employed to model societies whose members are to some extent cooperative toward each other. To achieve better results via cooperation, agents must be able to reason about their own belief states, and those of others. They must also be able to reason about what a group of agents can do, because it is often the case that a group can fulfill objectives that are out of reach for a single agent. Borrowing from Philosophy, agents should be endowed, at least to some extent, with a "Theory of Mind" [4], which refers to the cognitive capacity to attribute mental states to self and others. Many kinds of logical frameworks appeared in the literature which try to emulate cognitive aspects of human beings, also from the cooperative point of view [5]. In this paper we propose a particular logical framework, *L-DINF*, that draws inspiration from the concepts of Theory of Mind and of Social Intelligence. We consider the notion of executability of agents' inferential actions that may, moreover, require resource consumption. Hence, in order to execute an action the agent must possess the necessary *budget*. In our approach, when an agent belongs to a group, even if that agent does not have enough budget to perform an intended action, it may be supported by its group. So, 'our' agents are *aware* of themselves, of the group they belong, and possibly of other groups. Logics concerning some aspects of awareness have been proposed, starting from the seminal work of Fagin & Halpern [3]. Our proposal drew inspiration from some of the first works appeared in the literature, treating the logic of inference [6] and the logical system DES4$_n$ [2]. Differently from [2], in *L-DINF* inferential actions are represented both at the syntactic and semantic levels. Note that these actions are *mental operation*, not physical ones. However, the consequence of a mental operation can entail the execution of physical actions, among which the "active sensing" actions that an agent performs to check (aspects of) the state of its environment.

2 SYNTAX

Let $Atm = \{p, q, \ldots\}$ be a countable set of atomic propositions and Agt be a set of agents. A subset Atm_A of Atm represents the physical actions. The language of *L-DINF*, denoted by $\mathcal{L}_{L\text{-}DINF}$, is defined by the following grammar, where $p \in Atm$ and $i \in Agt$.:

$$\begin{array}{lcl} \phi, \psi & ::= & p \mid \neg\varphi \mid \varphi \wedge \psi \mid \mathbf{B}_i\, \varphi \mid \mathbf{K}_i\, \varphi \mid \\ & & do(\phi_A) \mid can\_do(\phi_A) \mid exec_G(\alpha) \mid [G : \alpha]\, \varphi \\ \alpha & ::= & \vdash\!(\varphi,\psi) \mid \cap(\varphi,\psi) \mid \downarrow\!(\varphi, \psi) \end{array}$$

The language of *inferential actions* of type α is denoted by $\mathcal{L}_{\mathrm{ACT}}$. The static part, *L-INF*, of *L-DINF* includes only those formulas not having sub-formulas of type α, namely, no inferential operation is admitted. The formula $do(\phi_A)$, where it is required that $\phi_A \in Atm_A$, denotes *actual execution* of action ϕ_A. Note that *do* is not axiomatized, as it represents what has been called in [7] a *semantic attachment*, i.e., a procedure which connects an agent with its external environment in a way that is unknown at the logical level. Moreover, $can\_do(\phi_A)$, where again $\phi_A \in Atm_A$, is a reserved syntax that allows an agent to refer to and reason about its own capabilities.

The formula $\mathbf{B}_i\, \varphi$ is read "the agent i explicitly believes that φ is true". Explicit *beliefs* are accessible in the *working memory* and are the basic elements of the agents' reasoning process. Instead, $\mathbf{K}_i$ is the well-known S5 modal operator; we use it to model/represent agent's *background knowledge* (in agent's *long-term memory*).

A formula of the form $[G : \alpha]\, \varphi$, with $G \in 2^{Agt}$, states that "φ holds after the inferential action α has been performed by at least one of the agents in G, and all agents in G have common knowledge about this fact". If an action is performed by an agent $i \in G$, the others agents belonging to the same group G have full visibility of this action and, therefore, as we suppose agents to be cooperative, it is as if they had performed the action themselves.

Borrowing from and extending [1], we distinguish three types of inferential actions α, that characterize the basic operations of forming explicit beliefs via inference:

- $\downarrow\!(\varphi, \psi)$: by performing this action, an agent tries to retrieve from her background knowledge the information that φ implies ψ and, if she succeeds, she starts to believe ψ;

Proc. of the 20th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2021), U. Endriss, A. Nowé, F. Dignum, A. Lomuscio (eds.), May 3–7, 2021, Online.

- $\cap(\varphi,\psi)$ is the inferential action of deducing $\varphi \wedge \psi$ from the explicit belief φ and the explicit belief ψ;
- $\vdash(\varphi,\psi)$ is the inferential action which consists in inferring ψ from φ in case φ is believed and, according to agent's working memory, ψ is a logical consequence of φ.

Finally, the formula $exec_G(\alpha)$ expresses executability. It has to be read as: "α is an inferential action that an agent in G can perform".

3 EXAMPLE

Consider a group of n agents, where each agent manages a smart home, which is a prosumer (producer+consumer) of energy. The energy budget available for the night is the difference between energy produced from the solar panel and energy consumed. Assume that agents are cooperative and that an agent i would like to use some appliance during the night, but its own budget is insufficient. Nevertheless, agent i could use the needed appliance if the group as a whole had sufficient budget. Let be $n = 4$ and assume that in world w_1 these four agents have the following budget to perform actions: $B(1, w_1) = 10, B(2, w_1) = 7, B(3, w_1) = 8, B(4, w_1) = 20$. The physical actions any agent can perform are: *switch-on–conditioner*, *switch-on–washing-machine*, and *close–electric-shutter*. As a prerequisite, to perform any physical action, agents have to complete some inferential process, composed of one or more inferential actions. Let us consider the following inferential actions

$\alpha_1 : \downarrow(temperature{-}high,\ switch\text{-}on{-}conditioner)$
$\alpha_2 : \downarrow(dirty{-}clothes,\ switch\text{-}on{-}washing\text{-}machine)$
$\alpha_3 : \downarrow(night \wedge thieves{-}fear,\ close{-}electric\text{-}shutter)$
$\alpha_4 : \cap(night,\ thieves{-}fear)$

whose costs are $C(i, \alpha_1, w) = 20$, $C(i, \alpha_2, w) = 12$, $C(i, \alpha_3, w) = 8$, $C(i, \alpha_4, w) = 1$ and such that $\alpha_j \in E(i, w)$ holds for each world w, each agent i, and each action α_j.

Assume that the knowledge base of each agent i contains rules:

1. $\mathbf{K}_i(temperature{-}high \rightarrow switch\text{-}on{-}conditioner)$
   That is, if an agent knows that the temperature inside the house is high, she can switch the conditioner on;
2. $\mathbf{K}_i(switch\text{-}on{-}conditioner \rightarrow close{-}electric\text{-}shutter)$
   That is, if an agent knows that someone switches the conditioner on, she can close the electric shutter to avoid the heat letting in from outside;
3. $\mathbf{K}_i(dirty{-}clothes \rightarrow switch\text{-}on{-}washing\text{-}machine)$
   That is, if an agent knows that there are dirty clothes in the washing machine, she can switch it on;
4. $\mathbf{K}_i(night \wedge thieves{-}fear \rightarrow close{-}electric\text{-}shutter)$
   That is, if an agent knows that it is night and someone has the fear of thieves, she can close the electric shutter.

Assume, moreover, that agents have the following beliefs:
$\mathbf{B}_1(temperature{-}high)$, $\mathbf{B}_2(dirty{-}clothes)$, $\mathbf{B}_3(thieves{-}fear)$, $\mathbf{B}_3(night)$, $\mathbf{B}_4(temperature{-}low \rightarrow switch\text{-}on{-}conditioner)$.

Note that, the latter formula —which means that if the temperature is low, then agent 4 can switch the (heater-)conditioner on—, represents an inference that agent 4 may perform by exploiting its working memory (i.e., its own beliefs). Namely, this implication allows agent 4 to infer $\mathbf{B}_4(switch\text{-}on{-}conditioner)$ if the belief $\mathbf{B}_4(temperature{-}low)$ is in its own working memory. Compare this formula with the formula (1) belonging to the knowledge base of the agent. In (1) the implication involves agent's long-term memory and the inference would exploit agent's background knowledge.

Suppose agent 1 wants to perform α_1. She alone cannot perform α_1 because she does not have enough budget. But, using the inferential action $[G : \downarrow(temperature{-}high, switch\text{-}on{-}conditioner)]$, with $G = \{1, 2, 3, 4\}$, the other agents can lend her part their budgets to share the cost. The action can be performed by the group G, because $\frac{C(1,\alpha_1,w_1)}{|G|} \leq \min_{h \in G} B(h, w_1)$. Hence, agent 1 can infer $\mathbf{B}_1(switch\text{-}on{-}conditioner)$. This translates into the execution of the concrete physical action. Notice that, since the inferential action is considered as performed by the whole group G, each agent in G adds $\mathbf{B}_i(switch\text{-}on{-}conditioner)$ to her beliefs. After the execution of the action the budget of each agent is updated as well. The new budgets are: $B(1, w_2) = 5, B(2, w_2) = 2, B(3, w_2) = 3, B(4, w_2) = 15$ where, for simplicity, we named w_2 the situation reached after the execution of the action. Let us now consider the case in which, in such situation, agent 2 wants to perform *switch-on–washing-machine*, enabled by the inferential action $\downarrow(dirty{-}clothes, switch\text{-}on{-}washing\text{-}machine)$. In this case, the right precondition $\mathbf{B}_2(dirty{-}clothes)$ holds, but, even considering the entire group G, the available budgets do not satisfy the constraint $\frac{C(2,\alpha_2,w_2)}{|G|} = 3 \leq \min_{h \in G} B(h, w_2)$ (in particular, because the available budget of agent 2 is 2). Let us, instead, assume agent 3 wants to perform α_3 (in situation w_2), in order to enable the physical action *close–electric-shutter*. This cannot be done directly, because before executing the inferential action

$$\downarrow(night \wedge thieves{-}fear, close{-}electric\text{-}shutter),$$

she has to perform $\cap(night, thieves{-}fear)$ in order to infer the belief $\mathbf{B}_3(night \wedge thieves{-}fear)$. Considering the current budget, the execution of $[\{3\} : \cap(night, thieves{-}fear)]$ can be completed (and, after that, the budget for agent 3 becomes 2). In this manner agent 3 obtains the belief needed as precondition to the execution of $\downarrow(night \wedge thieves{-}fear, close{-}electric\text{-}shutter)$. Nevertheless, in order to execute such action she needs the help of other agents (because her budget does not suffice). If all agents contribute, the new belief $\mathbf{B}_3(close{-}electric\text{-}shutter)$ is inferred through $[G : \downarrow(night \wedge thieves{-}fear, close{-}electric\text{-}shutter)]$. Again, all agents in G acquire the belief inferred by agent 3 and extend their beliefs.

4 CLOSING DISCUSSION

In this paper we discussed some cognitive aspects of autonomous systems, concerning executability of actions in a group of agents according to the available (collective) budget. Agents are supposed to be cooperative in order to achieve collectively goals otherwise unattainable by a single agent. To model these aspects we proposed the new modal logic *L-DINF*. Space constraints prevented us from describing its formal semantics and from introducing its axiomatization, for which the authors obtained both soundness and strong completeness results. Consequently, we restricted ourselves to presenting a significant example that practically demonstrates the benefits of the new logical framework.

In future work we mean to extend our logic to integrate temporal aspects, i.e., in which instant or time interval an action has been or should be performed, and how this may affect the functioning of agents and groups.

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