HEAT CONDUCTION IN ONE DIMENSION

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ABSTRACT

This paper establishes a solution for one dimensional conduction of heat in non-homogeneous materials. It also depicts how one dimensional conduction of heat is used in various appliances and different areas. It shows the importance of the same, the heat equation derivation and the system of equations are established in a very concise way. Some sample examples are provided to show the applicability and effects of one dimensional steady conduction of heat.

KEYWORDS : Heat conduction, Heat transfer, Steady state conduction, Unsteady state conduction

INTRODUCTION

Conduction is the transfer of energy from the more energetic particles of a substance to the adjacent less energetic ones as result of interactions between the particles. The term 'one-dimensional' is used with to heat conduction problem when:

1. Only one space coordinate is required in order to describe the temperature distribution within a heat conducting body;
2. Edge effects are being neglected;
3. The flow of heat energy always takes place along the coordinate measured normal to the surface.

This paper depicts the applications of one dimensional conduction of heat in various areas, a survey performed in order to properly understand how one dimensional conduction of heat takes place and what the results of it are.

EQUATION METHODOLOGY

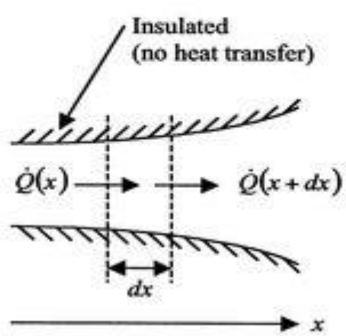

Fig 1.0 : Heat transfer in an insulated pipe

For one-dimensional heat conduction (temperature depending on one variable only), we can devise a basic description of the process. The first law in control volume form (steady flow energy equation) with no shaft work and no mass flow reduces to the statement that for all surfaces

$$\sum Q = 0$$

(no heat transfer on top or bottom of Figure1.0). The heat transfer rate in at the left (at x) is

$$Q(x) = -k(A)\{\frac{dT}{dx}\} \qquad (1.1)$$

The heat transfer rate on the right is

$$Q(x+dx) = Q(x) + \left\{\frac{dQ}{dx}\right\}|_x dx + \cdots \qquad (1.2)$$

Using the conditions on the overall heat flow and the expressions in (1.1) and (1.2)

$$Q(x) - \left[Q(x) + \left\{\frac{dQ}{dx}\right\}(x)dx + \cdots\right] = 0 \qquad (1.3)$$

Taking the limit as dx approaches zero we obtain

$$\left\{\frac{dQ(x)}{dx}\right\} = 0 \qquad (1.4)$$

or

$$\frac{d}{dx}\left(kA\left\{\frac{dT}{dx}\right\}\right) = 0 \qquad (1.5)$$

If k is constant (i.e. if the properties of the bar are independent of temperature), this reduces to

$$\frac{d}{dx}\left(A\left\{\frac{dT}{dx}\right\}\right) = 0 \qquad (1.6)$$

or (using the chain rule)

$$\{\frac{dT}{dx}\}^2 + \left\{\left(\frac{dA}{Adx}\right)\left(\frac{dt}{dx}\right)\right\} = 0 \qquad (1.7)$$

Equation (1.6) or (1.7) describes the temperature field for quasi-one-dimensional steady state (no time dependence) HEAT TRANSFER.

Example: Heat transfer through a plane slab

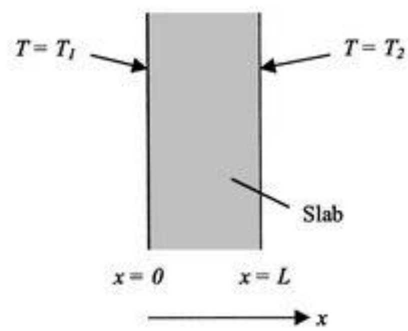

Figure 2: Temperature boundary conditions for a slab

For this configuration (Figure 2), the area is not a function of x , i.e. A=constant . Equation (1.7) thus becomes

$$\frac{d^2t}{dx^2} = 0 \quad (1.8)$$

Equation (1.8) can be integrated immediately to yield

$$\frac{dT}{dx} = a \quad (1.9)$$

and

$$T=ax+b \quad (1.11)$$

Equation (1.11) is an expression for the temperature field where a and b are constants of integration. For a second order equation, such as (1.8), we need two boundary conditions to determine a and b . One such set of boundary conditions can be the specification of the $T(0)=T_1$ $T(L)=T_2$ temperatures at both sides of the slab as shown in Figure 2, say ; .

The condition $T(0)=T_1$ implies that $b=T_1$. The condition $T_2=T(L)$ implies that $T_2=aL+T_1$, or $a = \frac{(T_2 - T_1)}{L}$

. With these expressions for a and b the temperature distribution can be written as

$$T(x) = T_1 + \{\frac{(T_2 - T_1)}{L}\}x \quad (1.12)$$

This linear variation in temperature is shown in Figure 3 for a situation in which $T_1 > T_2$

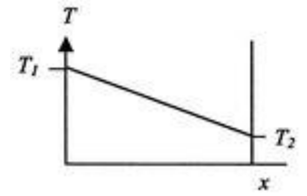

Figure 3: Temperature distribution through a slab

The heat flux { q } is also of interest. This is given by

$$q = -k\left\{\frac{(T_2 - T_1)}{L}\right\} = constant \tag{1.13}$$

LITERATURE REVIEW

Steady and unsteady state conduction:

- When the temperature difference resulting in conduction is constant , steady state conduction takes place. Hence, the spatial distribution of temperatures in the conducting object does not change any further.
- During any period in which temperatures are changing in time at any place within an object, the mode of thermal energy flow is termed transient conduction or non-steady state conduction.

Heat Conduction Experiment

Heat conduction(ALSO COVERING ONE DIMENSIONAL HEAT CONDUCTION) is when heat is transferred through molecular agitation without any movement of the object as a whole. Essentially, as a molecule heats up, it moves and shakes quickly, then moves the other nearby molecules, which move and shake in turn. Bit by bit, heat is transferred through those moving', shaking' molecules in a chain reaction. This may seem complicated, but it's simple to demonstrate with this heat conduction experiment.

Start by putting a pot of water on the hot stove. Once the water is good and hot (boiling or near boiling), carefully place 3 different spoons in the pot – one metal, one plastic or rubber, and one wooden spoon.

Fig 1. THREE SPOONS IN A POT WITH BOILING WATER

Fig 2. STAGE TWO SHOWING RATE OF ONE DIMENSIONAL CONDUCTION OF HEAT

The butter on the metal spoon almost immediately melted away; the butter on the wooden spoon melted some; meanwhile, the butter on the plastic spoon stayed firm much longer. That's heat conduction at work. The heat was transferred by moving molecules within the spoons. The spoons themselves didn't move, but their molecules did. This experiment also gives us some insight into what kinds of materials conduct heat well.

DATA SURVEY GRAPH

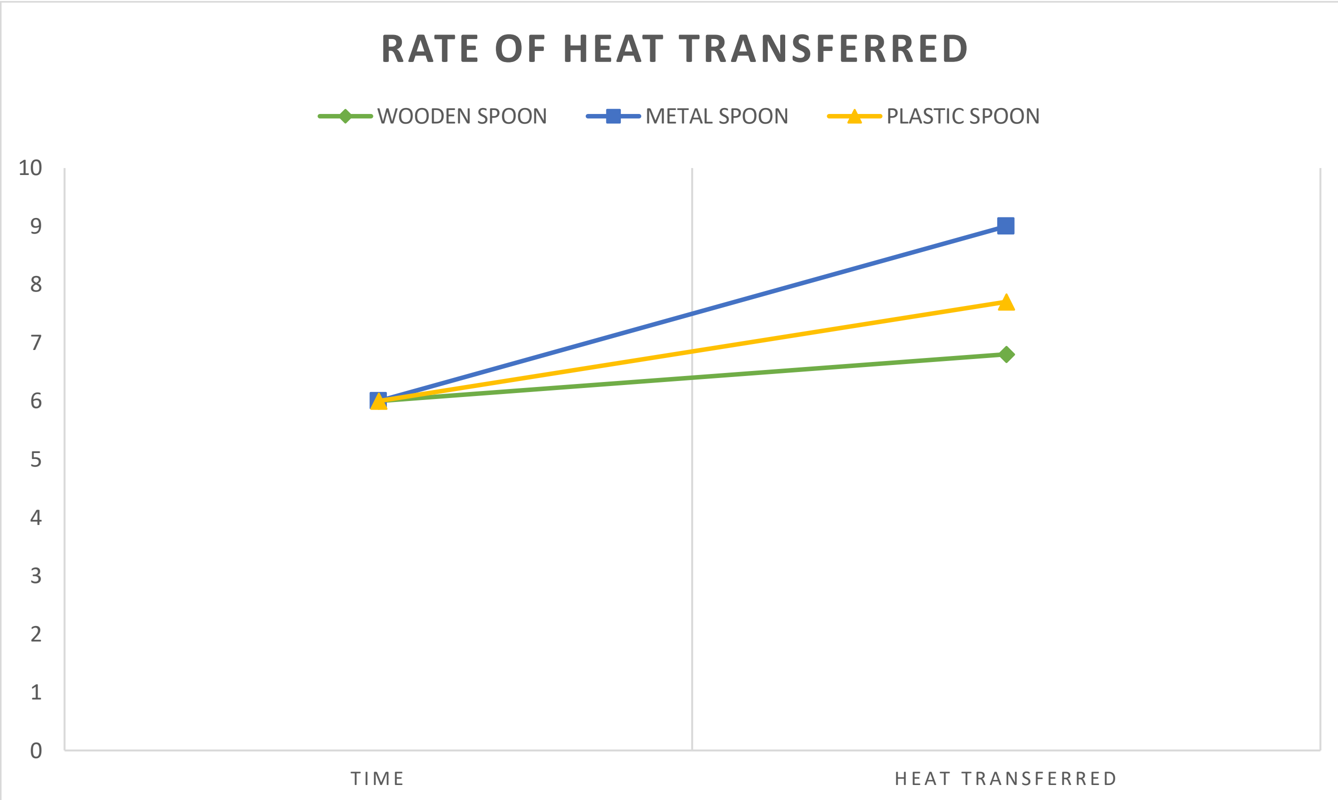

Fig (a) : GRAPH DEPICTING RATE OF HEAT TRANSFERRED IN 3 DIFFERENT MATERIALS.

PROPOSED ENHANCEMENTS

An "approximate" solution of a realistic model of a physical problem is usually more accurate than the "exact" solution of a crude mathematical model. When attempting to get an analytical solution to a physical problem, there is always the tendency to oversimplify the problem to make the mathematical model sufficiently simple to warrant an analytical solution. Therefore, it is common practice to ignore any effects that cause mathematical complications such as nonlinearities in the differential equation or the boundary conditions. So it comes as no surprise that nonlinearities such as temperature dependence of thermal conductivity and the radiation boundary conditions are seldom considered in analytical solutions. A mathematical model intended for a numerical solution is likely to represent the actual problem better. Therefore, the numerical solution of engineering problems has now become the norm rather than the exception even when analytical solutions are available.

CONCLUSION

Interactions between matter and energy is in everything from the microscopic jiggling of atoms to the gargantuan collisions of galaxies. Understanding the universe depends on becoming familiar with how matter responds flow of energy, it is evident that many physical phenomena can be modelled using partial differential equations in particular heat transfer. In many cases analytical solutions are not enough thus we rely on numerical solutions to obtain more information on the problems .In this paper we have observed that the application of numerical methods are limited to the cases where the functions under consideration are well behaved. To have a general way of solution we have to find new methods of discretizing the boundary conditions so as we can get solutions that are in line with the experimental results. Thus we need to undertake more research on this topic to further our knowledge so that we can effectively utilize our limited resources for the betterment of humanity.

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