The Flexible Socio Spatial Group Queries

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ABSTRACT

A socio spatial group query finds a group of users who possess strong social connections with each other and have the minimum aggregate spatial distance to a meeting point. Existing studies limit to either finding the best group of *a fixed size* for a single meeting location, or a single group of a fixed size w.r.t. multiple locations. However, it is highly desirable to consider multiple locations in a real-life scenario in order to organize impromptu activities of *groups of various sizes*. In this paper, we propose *Top k Flexible Socio Spatial Group Query* (*Top k-FSSGQ*) to find the top k groups w.r.t. multiple POIs where each group follows the minimum social connectivity constraints. We devise a ranking function to measure the group score by combining social closeness, spatial distance, and group size, which provides the flexibility of choosing groups of different sizes under different constraints. To effectively process the *Top k-FSSGQ*, we first develop an *Exact* approach that ensures early termination of the search based on the derived *upper bounds*. We prove that the problem is NP-hard, hence we first present a heuristic based approximation algorithm to effectively select members in intermediate solution groups based on the social connectivity of the users. Later we design a *Fast Approximate* approach based on the *relaxed social and spatial bounds, and connectivity constraint heuristic*. Experimental studies have verified the effectiveness and efficiency of our proposed approaches on real datasets.

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1. INTRODUCTION

Various social network sites now allow users to capture their locations through GPS-enabled devices and share them through check-ins or mentions in posts. As a result, socio spatial networks are emerging where each user is associated with a physical location along with the connectivity with other members of the network. Given such a network, socio spatial group queries [7, 25, 30, 33] aim to find the 'best' group against a Point Of Interest (POI) where the users possess social tightness within the group and have spatial closeness to the POI. An example of such queries is to find a group of three members, who are located close to a particular restaurant and socially connected to at least one of the other members, so that the group is competent for a targeted advertisement of a "20% discount for a table of three" offer running at that restaurant.

Although existing works have contributions towards finding an important class of group queries, there exists several gaps with the real-life applications, particularly the following major limitations:
(i) *Impracticality of specifying group size:* In each of the existing socio spatial group queries ([7, 25, 30, 33]), a single value as the size of the group (i.e., fixed size group) needs to be specified by the query issuer a priori. However, without prior knowledge of the social connections and users' locations, it is difficult to provide an exact and explicit size for the desired group. For example, "buy two get one" is a traditional offer. However, the advertiser may find that most of the groups close to the POI are generally of four people. So the group size of three members may not be a feasible offer.
(ii) *Finding only the best group for only one POI:* The algorithms in [7, 30, 33] can find only one group against only one query POI, where the solutions are not easily extendable for multiple groups or POIs (Section 2). Finding multiple groups are important for advertisers, and multiple POIs are important to suggest the best meeting location. For example, given multiple event locations of a festival, each resulting group can get advertisement for its nearest event. To the best of our knowledge, the only existing work that incorporates multiple POIs is [25]. However, their proposed algorithm can find **only one** group (i.e., $k = 1$) as the result. They use an R-tree and a Ball-tree as index, and the algorithm is not easily extendable for $k > 1$, which limits the applicability of the work.
(iii) *Profit optimization - a trade-off between the group effectiveness and the group size:* Advertisers want to offer the best deal to attract closely connected users who are located nearby to the POI; and at the same time they want to maximize their profit by preferring a larger group, as increasing the group size is more profitable. However, increasing the group size may decrease the users' satisfaction in the meet-up as it increases the chance of meeting with more unknown people. Thus, there should have a trade-off between satisfaction and cost; and finding the optimal group size is essential for such scenarios. In literature, the group score is generally defined as the weighted combination of social and spatial scores of the group, but the size of the group is ignored in the scoring function. Hence, the existing work are not suitable to find the balance between the group effectiveness and group size trade-off.

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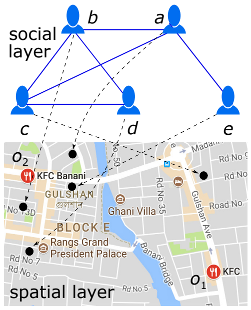


User	o_1	o_2
a	9	3
b	12	2
c	6	8
d	9	5
e	8	3

Figure 1: A socio spatial graph, the table shows the distances of the users from each meeting point

To address the above limitations, we propose a novel Top-k Flexible Socio Spatial Group Query (*Top k-FSSGQ*) that finds top k groups with flexible group sizes for a given set of POIs (meeting points). Consider a scenario, where a chain-shop has multiple branches and they want to find k best groups to offer total k coupons in different locations. Since the owner does not have any prior knowledge on the groups and users around each branch, she cannot decide on the number of coupons required in different branches, and what should be the formation of the coupons, e.g., "buy two get one" or "buy three get one". Thus the objective is to find top k groups in total w.r.t. any query POIs (branches) with the highest socio spatial ranks, where the group size can vary within a query minimum and maximum size, specified by the query issuer based on the application. For example, if the POI is a restaurant, its maximum table size can be the maximum allowable group size. *Formally, for a given socio-spatial graph, the minimum and maximum number of users allowed in a group, a minimum acquaintance constraint, a set of candidate meeting points, and a maximum distance constraint, the Top k-FSSGQ query returns k best groups and their corresponding meeting points, where each group satisfies the minimum acquaintance and group size constraints, and the location of each member satisfies the distance constraint. We also guarantee that a top-k group cannot be fully contained in another top-k group*, for example, given a set of users $\{a, b, c, d, e\}$, then groups like $\{a, b, c\}$ and $\{a, b\}$ both cannot be in the result.

We now present an example to illustrate the *Top k-FSSGQ* query, which also highlights the limitations of the existing studies.

EXAMPLE 1. *Figure 1 presents a graph of users $V = \{a, b, c, d, e\}$, where the upper and lower part represent the social and spatial layer, respectively. Each edge in the social layer represents the connectivity between two users. Each user has a location, shown with an arrow from the social to the spatial layer. Here, $O = \{o_1, o_2\}$ is a set of POIs. Let, we want the top 2 groups with group size minimum 3 and maximum 4, and the resultant groups should satisfy the minimum acquaintance constraint, 1.*

As the existing studies [7, 30, 33] can only find the best group of only one fixed size for only one POI, their algorithm needs to be repeatedly reissued for each of the POIs and for each of the group sizes between the minimum and maximum values. Although Shen et al. [25] allows multiple POIs as the query input, they can only find one single group (i.e., top-1 group) of a fixed size. Therefore, their approach cannot be directly applied to find the top-2 groups. From this example, it is evident that, (i) If the group size is strictly fixed, potential profitable groups of other sizes may get excluded, (ii) A high computational cost is incurred for the existing approaches by reissuing query with different sizes for different POIs. Here, our aim is to find the top-2 groups ($\{b, a, d, c\}$ and $\{b, a, e, c\}$ in this example, detailed calculations are shown in Section 5.2) efficiently by avoiding the repeated unnecessary calculations.

To process *Top k-FSSGQ* query, we extend the approach presented in [30] as our baseline. In this baseline, we repeat the approach for all possible groups of size between the minimum and maximum value against all meeting points that satisfy the constraints, rank the groups and then return the top k groups as result. As the baseline requires a high computational cost (details in Section 3), we propose multiple efficient solutions: (i) an efficient *Exact* approach that finds optimal groups with much less computation overhead, (ii) a heuristic based *Approximate* approach which further improves the efficiency by selecting members of the groups based on the social connectivity, (iii) a *Fast Approximate* approach that answers the query much efficiently by sacrificing the quality of the groups slightly, and (iv) a *Greedy Approximate* approach.

The key idea of the approaches is to derive *theoretical bounds on spatial distance and social connectivities* to effectively prune a large number of candidate groups that cannot be an answer. In details, we expand our search for all meeting points in parallel and select users for possible solution group for each POI. Selection of users is processed by prioritizing both spatial and social aspect so that the formed groups can satisfy the acquaintance constraint and possess the minimum aggregate spatial distance. For *Exact* approach, we exploit the connectivity and locations of already fetched users w.r.t. each meeting point to derive *upper bounds* that can safely determine whether we need to further explore the space for a higher rank group. We also define a bound for the *familiarity constraint* for a user that must be satisfied in order to qualify the group as an answer for our heuristic based *Approximate* approach. For the *Fast Approximate* approach, we design more powerful pruning strategies to reduce the exploration. We develop *an upper bound on spatial distance and a lower bound on social connectivity* of a *member* (in contrast to all members in the exact approach). Based on these bounds, we develop an early terminate strategy.

The contributions of this paper are described as follows.

(i) We re-define the socio spatial group query for multiple meeting points, and design a flexible ranking function consisting of social cohesiveness, spatial closeness, and group size.

(ii) We develop an *Exact* approach which is significantly faster than the baseline and multiple *Approximate* approaches that are highly efficient with almost similar quality of result.

(iii) We develop an early termination strategy and several pruning rules for improving the efficiency of the *Exact* approach and the *Approximate* approach based on the upper bound of spatial distance and lower bound of social connectivity.

(iv) We conduct extensive experiments to verify the efficiency of our developed algorithms and effectiveness of our approximate solutions by using real datasets.

2. RELATED WORK

Different variants of group queries in social network have been studied in literature recently. We can categorize the queries based on (i) group with social connectivity constraints only; (ii) group based on spatial distance; and (iii) group with both constraints.

2.1 Group Queries by Social Connectivity

Social connectivity based group queries can further be classified as *team formation* [16, 18], *community detection* [9, 22], and *community search* [5, 6, 8, 12, 19, 26]. *Team formation* [16, 18] aims at finding a group of experts in a social graph with required skills while minimizing communication cost within that group. *Community detection* aims at finding all communities in a given graph based on specific criteria such as modularity [15] or the other different contexts [9, 22]. Whereas, given a query node, *community search* aims at finding a group (community) of nodes in a large graph where the resulting community must contain the query node. Here, the query can be a single node [5–8, 12, 19, 26, 33] or multiple nodes [26] in a graph. *Minimum degree metric* or *k-core* is often being used as the social constraint in defining communities [6, 7, 26, 33]. Besides, *k-clique* [5], *k-truss* [12], and *connectivity* [11] have also been considered in online community search. In addition to the social factor, a socio temporal group query [29] emphasizes the availability of all attendees in an impromptu activity.

2.2 Group Queries Based on Spatial Distance

Group nearest neighbor queries that find a meeting point with the smallest aggregate distance (summation, maximum, etc.) from the group have been extensively studied in different contexts [2, 3, 20, 23, 31]. The studies [14, 32] explored optimal location query for a group, where given a set of users and a set of POIs, the query finds the location of a new meeting point that minimizes the average distance from each user to the closest meeting point [14, 32]. Other similar works [13, 24, 27, 28] find a location for a new server such that the maximum distance between the server and any client is minimized. Papadias et al. [23] find a location that minimizes the sum of the distances from the users. Ali et al. [3] find optimal subgroups and the meeting point for each subgroup that minimizes the aggregate spatial distance for the subgroup; whereas Ahmed et al. [2] extend [3] to include both spatial proximity and keyword similarity while selecting a meeting point for a subgroup.

2.3 Socio Spatial Group Queries

For a given socio spatial graph, Yang et al. [30] propose a socio spatial group query (SSGQ) that finds a set of members against a fixed rally point where the aggregate spatial distance between members and the rally point is minimized and each member is allowed to be unfamiliar with at most a maximum number of members in that group. Members' locations are indexed using an *R-tree*, and new members are added to an initially empty set based on distance ordering and familiarity checking. Finally, a resultant group of predefined fixed size is returned. Our work is different from this work in several aspects. Our aim is to find top k groups of variable size for multiple POIs and each member has minimum level of acquaintance with the other members. In contrast, SSGQ only considers one rally point, which is impractical as for a large socio spatial graph multiple rally points may exist. Moreover, the average minimum familiarity constraint [30] cannot always be preferable for an individual where that member possesses weak social connectivity within the resulting group. Lastly, potential candidate members may be excluded due to the fixed size group.

Shen et al. [25] propose the multiple rally-point social spatial group query (MRGQ) that chooses a suitable rally point from the multiple points and the corresponding best group, which exhibits the minimized spatial distance between group members to the best rally point. The resulting group is of fixed size and satisfies the average minimum familiarity constraint. To efficiently process the query, an *R-tree* is used to index member locations and a *Ball-tree* is used to index rally points. As only the best group is returned, indexing both activity locations and member locations makes solution efficient. Incorporating the idea of multiple rally points enhances the acceptance of MRGQ, but finding only the best group for multiple POIs limits the applicability of socio spatial group queries. The MRGQ also considers an average minimum social connection as social cohesiveness, which limits individual's satisfaction. Moreover, the resulting group has fixed size limitation.

Zhu et al. [33] propose a new class of geo-social group queries with minimum acquaintance constraint (GSGQs), where the result group guarantees the worst-case acquaintance of all users. GSGQs takes three parameters: *query issuer*, *spatial constraint*, and *social constraint*. *Query issuer* is a member in the given graph. Minimum degree c is the *social constraint*. GSGQs considers three different spatial constraints, i.e., GSGQs returns largest c-core within a range or a c-core of more than a fixed size or a c-core of a fixed size. Fang et al. [7] propose a spatial-aware community (SAC), which is a connected c-core where the members in the resulting group are located within a spatial circle having a minimum radius. SAC also maintains the minimum acquaintance constraint. c-core is experimented effectively [7, 33] when a single query issuer is a member in resulting group. However, in a large socio spatial graph, different sized groups with different social and spatial configuration exist. Socio spatial query issued by a member [7, 33] serves individual's purpose but does not help to understand the presence of various socio spatial clusters in a given graph.

Armenatzoglou et al. [4] propose a set of geo-social ranking (GSR) functions to combine both social and spatial factors and find top-k users with respect to each of these functions. They introduce a general GSR framework and propose four functions that cover several practical scenarios. In our solution, we have adopted similar technique like [4] to score social tightness and spatial closeness of a group. Although GSR functions help understanding basic ranking criteria in assessing the score of social and spatial features, ranking users cannot serve the goal of socio spatial group queries. For this reason, we are computing the score of groups rather than users which helps to find top k groups in a given socio spatial graph.

3. PROBLEM FORMULATION & BASELINE

Let a socio-spatial graph be $G = (V, E)$ where V is the set of members and E is the set of edges representing the social connections. Let l_v be the location for $v \in V$ and O be a set of candidate meeting points. We will first define our socio-spatial group score function. A group with strong social connection among members and less aggregate spatial distance to a meeting point is preferred, where a user is interested in meeting points within a spatial range. Group size is also important as a large group with the same connectivity and aggregate spatial aspect is more preferable. We adopt the social and spatial score measures from [4]. Now consider a sub graph of G as $G' = (V', E')$ and a meeting point $o' \in O$. The socio-spatial group score of (G', o') can be measured as follows.

DEFINITION 1. *(Social connectivity score) The social score is computed based on the average social connectivity, provided that each member satisfies a minimum acquaintance constraint, c. The social connectivity score S_{sc} is the density of G' and computed as $\frac{2|E'|}{|V'|(|V'|-1)}$.*

DEFINITION 2. *(Spatial closeness score) The spatial closeness score S_{sp} of G' is inverse to the normalized average distance of the group members to o' and computed as $1 - \frac{\sum_{v_i \in V'} d(v_i, o')}{d_m |V'|}$ where $d(v_i, o')$ is the spatial distance from v_i to o', and d_m is the maximum spatial distance of a user from a meeting point.*

DEFINITION 3. *(Group size score) The group size score S_{gs} of G' is directly proportional to its group size $|V'|$ and is computed as $\frac{|V'|}{n''}$, where n'' is the maximum group size.*

DEFINITION 4. *(Socio-spatial group score) Given a subgraph $G' = (V', E')$ in G, a meeting point $o' \in O$, a maximum group* *size n'' and a maximum spatial range d_m, we measure the socio spatial group score $S(G', o')$ as a linear weighted combination of the individual scores, which is the most common type of combination used in the literature [4, 21]. Note that, any other types of combinations of the scores can also be used in our solutions.*

$$S = \alpha * S_{sc} + \beta * S_{sp} + \gamma * S_{gs} \quad (1)$$

Here, $\alpha + \beta + \gamma = 1$, and the values of α, β and γ can be set based on priority of social, spatial, and group size, respectively. Based on the socio-spatial group score $S(.,.)$, the *Top k-FSSGQ* query can be formalized as follows.

DEFINITION 5. *(Top k-FSSGQ) Given a socio spatial graph $G = (V, E)$, a set O of locations as the meeting points, a minimum group size n', a maximum group size n'', a minimum acquaintance constraint c, a maximum spatial range d_m, and a parameter k, the Top k-FSSGQ finds a ranked list of top-k groups and their corresponding meeting points from O, each as a tuple of the form $(G_i, o_i, S(G_i, o_i))$. Here, each G_i is a subgraph of G, and each $(G_i, o_i, S(G_i, o_i))$ must satisfy the following conditions.*

(1) (G_i, o_i) is considered as an eligible candidate if and only if it meets the spatial and social constraints, i.e., $n' \leq |V(G_i)| \leq n''$ holds, the minimum degree of any node in G_i is no less than c, and the maximum distance of any user in G_i to o_i is no larger than d_m.

(2) $S(G_i, o_i) \geq S(G_{i+1}, o_{i+1})$ for any $1 \leq i \leq k-1$;

(3) $\nexists o \in O$, $G_j \subseteq G$ and $S(G_j, o) > S(G_i, o)$ where $1 \leq i \leq k$, $j > k$ and (G_j, o) is an eligible candidate.

Parameter settings and choice of default values. *The problem formulation is made generalized using different parameters to cater for different needs of groups. As a component of the scoring function is the group size, the group size constraints n' and n'' are optional. Here, n' can be set as default to '1', and n'' can be set to the number of users of the social graph. Similarly, the minimum acquaintance (c) and the maximum distance (d_m) are useful to filter out the groups that the user does not want in the result. If the user is unsure of these values, c can simply be set to default '1' and d_m be infinity so the constraints do not have any affect on the result. As value of all of the components in Equation 1 is normalized between [0,1], γ can be replaced with $(1-\alpha-\beta)$ without losing generality.*

THEOREM 1. *The Top k-FSSGQ problem is NP-hard.*

PROOF. We prove this by the reduction from n'-clique. Given a graph G_c, n'-clique decision problem determines whether the graph contains a clique, i.e., a complete graph of n' vertices. For Top k-FSSGQ problem, assume that $G = G_c, c = n'-1, n'' = n'$, $d_m = \infty$, $O = \{o\}$ and $\forall v \in V, d(v, o) = 1$. We first prove the necessary condition. If G_c contains a n'-clique, there must exist a group with the same vertices in the n'-clique such that every member has social connectivity with all the other members in that group. Hence the total spatial distance is n'. We then prove the sufficient condition. If G in Top k-FSSGQ has a group of minimum size of n' and maximum size of n'' and $c = n' - 1$, G_c in problem n'-clique must contain a solution of size n' too. Therefore, the Top k-FSSGQ is an NP-hard problem. □

Baseline approach: We develop a baseline by extending one of the most relevant work proposed by of Yang et al. [30]. Yang et al. developed a sub-optimal solution for a socio spatial group query that finds the best group of a *fixed* size against a *single* meeting point, where the resultant group follows the required *average* minimum acquaintance constraint among members. Thus to find the answer of Top k-FSSGQ, for each meeting point, we find the best groups for each allowable group size (between the minimum and maximum group size), where the groups follow our required minimum acquaintance constraint c. Then, we rank all the groups that are found w.r.t. all meeting points to find the top k groups. Note that in the above steps we only consider members that fall within the maximum spatial range d_m.

Table 1: Basic notation

Symbol	Description
$d(v, o)$	The spatial distance between $v \in G$ and a $o \in O$
n' (n'')	Minimum (maximum) query group size
c	Minimum acquaintance constraint
d_m	Maximum spatial distance constraint
k	No. of results to be returned
α, β, γ	Preference parameters in the scoring function
V_{I_q} (V_{R_q})	The set of already explored members (remaining members) of a candidate group for a $o_q \in O$
f_c (f_k)	Current social connectivity in V_{I_q} (k^{th} group)
d_c (d_k)	The current aggregate spatial distance of all members in V_{I_q} (k^{th} group)
δ_f (δ_d)	The additional increase of the total social connectivity (aggregate spatial distance)
f_m	The maximum additional social connectivity of new members to the group
$d_n^{\uparrow}$	Spatial upper bound for a group of size n
$maxdeg$	The maximum degree of the members in V_{R_q}
d_{min}	The minimum spatial distance of the members in V_{R_q} from the meeting point o_q
$f^{v\downarrow}$	The lower bound on social connectivity
$d^{v\uparrow}$	The upper bound on spatial distance
$f(v, V_{I_q})$	The number of social connectivity of v in V_{I_q}

4. AN EFFICIENT EXACT APPROACH

We propose an efficient exact solution for answering the Top k-FSSGQ. The key idea is to develop an early termination strategy based on our derived *upper bound on spatial distance* that avoids the exploration of a large number of groups. Next, we present our *advance termination strategy* that determines when we should stop the exploration of members w.r.t. a meeting point.

4.1 An Advance Termination Strategy

Let us assume that we have already found k initial groups that satisfy the necessary constraints of our query, and let $kth\_bestscore$ be the current score of the k^{th} group. Now we need to find whether any un-explored group has a higher score than the $kth\_bestscore$. Let for any meeting point $o_q \in O$, V_{I_q} be the set of already explored members for a candidate group and V_{R_q} be the set of remaining members that are yet to be explored. If we can guarantee that including more members from V_{R_q} to V_{I_q} will not yield any group having a score greater than $kth\_bestscore$, we can safely terminate the search process for the meeting point o_q. Based on the above observation, we will now formulate some bounds that can ensure the safe termination of the search.

Let f_k be the *total social connectivity* of the k^{th} group, n_k be the number of members in the group, and d_k be the aggregate spatial distance of the group members from the corresponding meeting point. Also, let f_c be the *current* total social connectivity of members in V_{I_q} and d_c be current aggregate spatial distance of all members of V_{I_q} from the meeting point o_q. The ranking score of a group is computed based on the weighted combination of three scores: social score, spatial score and group size score. Thus, we first determine the *score gains* that can be obtained by the new group w.r.t. the k^{th} best group in these three scoring measures.

Social score gain: The social score of the k^{th} group is $f_k/(n_k * (n_k - 1))$ (Definition 1). Let, we are considering some more members to be included in the currently explored group V_{I_q} so that final group size becomes n where $n' \leq n \leq n''$. In this process, let δ_f be the additional increase of the total social connectivity if we add more members to V_{I_q} from V_{R_q}. Then, the social score gain of any newly formed group w.r.t. the social score of the k^{th} group is:

$$\Delta S_{sc} = \frac{f_c + \delta_f}{n * (n-1)} - \frac{f_k}{n_k * (n_k - 1)}$$

Spatial score gain: Similarly, the spatial score of the k^{th} group is $1 - d_k/(n_k * d_m)$ (Definition 2). Let δ_d be the additional increase of the aggregate spatial distance of the currently explored group if we add more members to V_{I_q} from V_{R_q}. Thus, the spatial score of the newly formed group will be $1-(d_c+\delta_d)/(n*d_m)$. Hence the spatial score gain of the newly formed group from the spatial score of the k^{th} group is $\Delta S_{sp} = \frac{1}{d_m}\left(\frac{d_k}{n_k} - \frac{d_c + \delta_d}{n}\right)$

Group size score gain: Similarly, the gain of the group size score of the current group w.r.t. the k^{th} group (Definition 3) is as follows.

$$\Delta S_{gs} = \frac{n - n_k}{n''}$$

Based on the above formulations, we can derive the *combined score gain* of the new group over the k^{th} group. If the gain is positive, that implies that the new group may have a better score than the k^{th} group, thus a candidate for the result. Therefore, the new group must satisfy the following equation.

$$\begin{aligned}
Gain = \quad & \alpha * \Delta S_{sc} + \gamma * \Delta S_{gs} + \beta * \Delta S_{sp} > 0 \\
\Rightarrow \alpha & \left(\frac{f_c + \delta_f}{n*(n-1)} - \frac{f_k}{n_k*(n_k-1)}\right) + \frac{(n-n_k)*\gamma}{n''} \\
& + \frac{\beta}{d_m}\left(\frac{d_k}{n_k} - \frac{d_c+\delta_d}{n}\right) > 0 \\
\Rightarrow & \frac{\beta}{d_m}\left(\alpha\left(\frac{f_c + \delta_f}{n*(n-1)} - \frac{f_k}{n_k*(n_k-1)}\right) + \frac{(n-n_k)*\gamma}{n''}\right) \\
& + \frac{d_k}{n_k} - \frac{d_c+\delta_d}{n} > 0
\end{aligned}$$

$$\begin{aligned}
\Rightarrow Gain = n\Bigg(\frac{d_m}{\beta}\Bigg(\alpha\left(\frac{f_c + \delta_f}{n*(n-1)} - \frac{f_k}{n_k*(n_k-1)}\right) + \\
\frac{(n-n_k)*\gamma}{n''}\Bigg) + \frac{d_k}{n_k}\Bigg) - d_c - \delta_d > 0
\end{aligned} \tag{2}$$

Here, we have two unknown variables: additional total social connectivity δ_f and additional aggregate spatial distance δ_d that a group can achieve. To achieve the maximum gain in Eq. (2), we need to find the *maximum* possible δ_f and the *minimum* possible δ_d.

4.2 Computing δ_f

Let f_m be the maximum additional social connectivity that a new group can achieve if we include new members from V_{R_q} to V_{I_q} for $o_q \in O$. The next member from V_{R_q} to be included in V_{I_q}, can be connected with maximum $|V_{I_q}|$ members, and the second next member can be connected with maximum $|V_{I_q}|+1$ members, and so on. Thus, we get:

$$\begin{aligned}
f_m &= |V_{I_q}| + (|V_{I_q}|+1) + (|V_{I_q}|+2) + ... + (n-1) \\
&= \frac{(n - |V_{I_q}|) * (n + |V_{I_q}| - 1)}{2}
\end{aligned}$$

In this case, we assume that the maximum degree, $maxdeg$, of the *complete initial* set of members V_{R_q} for o_q is greater than or equal to $n-1$. However, since a member $v \in V_{R_q}$ cannot be connected with more than $maxdeg$ members, we can tighten the f_m bounds. Thus, we can derive f_m as follows.

$$\begin{aligned}
f_m = & |V_{I_q}| + (|V_{I_q}|+1) + (|V_{I_q}|+2) + ... + maxdeg \\
& + \underbrace{maxdeg + ... + maxdeg}_{(n-maxdeg-1) \quad times} \\
= & \frac{(maxdeg + |V_{I_q}|) * (maxdeg - |V_{I_q}| + 1)}{2} \\
& + (n - maxdeg - 1) * maxdeg
\end{aligned}$$

If the current candidate group V_{I_q} has already more than $maxdeg$ members, the next included members can be socially connected with at most $maxdeg$ members in V_{I_q}. Thus, when $maxdeg < n-1$ and $|V_{I_q}| > maxdeg$, then, $f_m = (n - |V_{I_q}|) * maxdeg$. A new edge in a socio spatial graph increases the social connectivity of the graph by two. Since f_m represents the maximum social connectivity that can be achieved by adding more members in V_{I_q}, the additional social connectivity of V_{I_q} can be increased by at most $2 * f_m$. Thus, we get $\delta_f = 2 * f_m$.

4.3 Distance Upper Bound

Based on the computed upper bound of δ_f, i.e., $\delta_f = 2 * f_m$, we can derive the distance upper bound, $d_n^{\uparrow}$, for the group by replacing the value of δ_f in Eq. (2) as:

$$\begin{aligned}
n\Bigg(\frac{d_m}{\beta}\Bigg(\alpha\left(\frac{f_c + 2*f_m}{n*(n-1)} - \frac{f_k}{n_k*(n_k-1)}\right) + \\
\frac{(n-n_k)*\gamma}{n''}\Bigg) + \frac{d_k}{n_k}\Bigg) - d_c = d_n^{\uparrow}
\end{aligned} \tag{3}$$

Hence, $d_n^{\uparrow}$ is the upper bound for the aggregate spatial distance of the members in V_{R_q} who can be included in V_{I_q} to form a feasible group of size n.

4.4 Advance Termination

Based on the computed $d_n^{\uparrow}$, we deduce an early termination strategy for the group search. Let d_{min} be the minimum spatial distance of the members in V_{R_q} from o_q. Let us assume that we want to form a new group of size n, where $n' \leq n \leq n''$. Thus, the minimum aggregate spatial distance of the new $n - |V_{I_q}|$ members can be computed as $d_{min} * (n - |V_{I_q}|)$. Hence, $\delta_d = d_{min} * (n - |V_{I_q}|)$. Consequently, the new group cannot be included in the answer list, if the following condition holds.

$$d_n^{\uparrow} \leq d_{min} * (n - |V_{I_q}|) \tag{4}$$

We can compute $d_n^{\uparrow}$ for each valid group size of n. If Eq. (4) holds for all values of n, we terminate the search for o_q as no better group is possible by adding new members to V_{I_q}. We formalize the above termination process in the following lemma.

LEMMA 1. *Let o_q be a meeting point, V_{I_q} be the list of members already included in the process of forming a group. If for $\forall n, d_n^{\uparrow} \leq d_{min} * (n - |V_{I_q}|)$, where $n' \leq n \leq n''$, then no group with a higher ranking score than the current k^{th} best group is possible by including more members to V_{I_q} , and thus the search can be terminated w.r.t. o_q.*

PROOF. Proof is omitted for the brevity of the presentation. □

Algorithm 1: Top k-FSSGQ$(G, O, n', n'', c, d_m, k)$

1 Initialize a global empty list L
2 Initialize a min priority queue Q
3 **foreach** *meeting point* $o_i \in O$ **do**
4   Initialize $V_{I_i} = \varnothing$
5    $V_{R_i} \leftarrow$`retrieveFromRtree`(o_i, d_m)
6    $V_{R_i} \leftarrow$`pruneUnqualifiedMembers`(V_{R_i}, c)
7   Q.push$(o_i, V_{I_i}, V_{R_i}, dist(V_{R_i}.get(0), o_i))$
8 `generateRankList`(Q, L, k)
9 **Procedure** `generateRankList`(Q, L, k)
10   **while** Q *is not empty* **do**
11       $o, V_I, V_R \leftarrow$ Q.pop()
12   **if** $|V_I| = n''$ *or* $|V_I| + |V_R| < n'$ **then**
13   continue
14   mark all members of V_R as *unvisited*
15   **while** $(|V_I| < n'')$ *and* $|V_R| > 0$ **do**
16   **if** *there is any unvisited member in* V_R **then**
17             $v, d_v \leftarrow$ `nextMember`(V_R, o_i)
18   **else** break
19   **if** `advanceTerminate`$(V_I, d_v, n', n'', L, k)$ **then** break
20          $flag, score \leftarrow$ `formGroup`(V_I, V_R, v)
21   **if** $flag = true$ **then**
22   **if** $|V_I| \geq n'$ *and* $score > L.get(k).score$ **then**
23   `updateRanklist`(L, V_I)
24   Q.push$(o, V_I, V_R, dist(V_R.get(0), o))$
25   Q.push$(o, V_I - \{v\}, V_R, dist(V_R.get(0), o))$
26   break;
27   **else**
28   mark v as visited

4.5 Algorithm

Algorithm 1 provides the pseudo code of the $Exact$ approach. Given a socio spatial graph G and the set O of meeting points, the $Exact$ approach for the Top k-FSSGQ returns a list L of top k groups ranked in increasing order of their socio spatial scores. The Top k-FSSGQ also takes the following input: the minimum n' and the maximum n'' allowable group size, the maximum spatial distance, d_m of members from a meeting point, and the minimum acquaintance constraint c. Here, the locations of the users are indexed with an R-tree [10].

For each meeting point o_i, we initialize an intermediate solution group V_{I_i} as empty and a remaining set V_{R_i} containing all members within d_m distance from o_i through `retrieveFromRtree` procedure (Lines 4-5). Members in V_{R_i} are sorted in ascending order of their distance to o_i. In Line 6, members whose social connectivity do not satisfy the acquaintance constraint c are excluded from V_{R_i} using `pruneUnqualifiedMembers` procedure.

The algorithm works in a best-first manner, where groups are formed by incrementally retrieving users w.r.t. different points. A priority queue Q is maintained, where each entity contains a tuple of a meeting point o_i, an intermediate group V_{I_i}, and remaining set of members V_{R_i} for o_i. Entities in Q are maintained in ascending order of the minimum spatial distance between o_i and locations of the members in V_{R_i}. We initially push the list of entities w.r.t. all meeting points in Q. Then, we call `generateRankList` procedure to find the desired groups and ranks.

In `generateRankList` procedure, the top entity (o, V_I, V_R) of queue Q is popped in each iteration. Then an inner loop starts that fetches the next unvisited member v from V_R, and check the feasibility of including v in V_I. The inner loop breaks when a member is successfully included in V_I. Procedure `advanceTerminate` checks whether exploration of V_I and V_R w.r.t. o can generate a group that can be in the final rank list L, and returns $true$ when V_I can be pruned in advance according to Lemma 1.

`formGroup` returns a pair $(flag, score)$. The flag is set to $true$ if $|V_I \cup \{v\}| < n'$, or $|V_I \cup \{v\}| \geq n'$ and group $V_I \cup \{v\}$ satisfies the acquaintance constraint. In either case v is included in V_I. In the later case, the score of the new group, $score$ is also returned. If $score$ is greater than the score of the k^{th} group, the rank-list L is updated. When $|V_I \cup \{v\}| \geq n'$ but the group $V_I \cup \{v\}$ does not satisfy the acquaintance constraint, the $flag$ is set to $false$ and $score$ is set to null. If $flag$ is $true$, for the next step of processing, two new entities are pushed into Q, where the first entity contains updated (o, V_I and V_R), the second entity contains the previous state, (o, $V_I - \{v\}$, V_R), i.e., before including v into V_I, which ensures the generation of other feasible groups excluding v.

4.6 Time Complexity

For each meeting point, the maximum number of entities in Q is $\mathcal{O}(2^{n''})$. The inner while loop (Lines 1.15-1.28) is executed at most $O(n'')$ iterations. The `nextMember`, `advanceTerminate`, and `updateRanklist` functions incur $\mathcal{O}(1)$, $\mathcal{O}(n'')$, and $\mathcal{O}(k)$ time, respectively. So the time complexity of `generateRankList` is $\mathcal{O}(|O|2^{n''}(n'' + |k|))$, where $|O|$ is number of meeting points. However, in practice, the bounds and the termination significantly reduce the number of entries in Q for each meeting point. Let C_{pre} be the initial filtering cost as shown in Lines 1.3-1.7, which includes object retrieval from R-tree and filtering out the users who do not satisfy familiarity constraints. Hence the total runtime of *exact* approach is $\mathcal{O}(|O|2^{n''}(n'' + |k|) + C_{pre})$.

5. HEURISTIC-APPROXIMATE APPROACH

In our *Exact* approach, we use distance upper bound based advanced termination strategy for pruning. However, since the problem is NP-hard, it may not be scalable for large datasets. Thus, we propose a *familiarity constraint satisfaction* heuristic function that calculates a lower bound on *connectivity* of each individual while considering as a potential group member. This heuristic further prunes a larger number of members based on connectivity constraint, which makes it a scalable solution.

5.1 Familiarity Constraint Satisfaction

In our *Exact* approach, for each meeting point, we include members in the intermediate group V_{I_q} from V_{R_q} in ascending order of spatial distance without considering their social connections. As a result, when a valid size group is formed, the group may not satisfy the minimum query acquaintance constraint. To overcome this problem, in the member inclusion process from V_{R_q} to V_{I_q}, we prioritize the users having strong social connectivity with the other members in V_{I_q}, and define a *familiarity constraint filtering function* to filter out groups that cannot be a result.

Let $|V_{I_q}| < n'$, and $f(v, V_{I_q})$ be the number of social connections that v already has with other members in V_{I_q}. If v needs to satisfy constraint c, v requires to have an additional $c - f(v, V_{I_q})$ connectivity. After including v in V_{I_q}, additional $n' - 1 - |V_{I_q}|$ members need to be included so that V_{I_q} becomes an n' size group. If $c - f(v, V_{I_q}) > n' - 1 - |V_{I_q}|$, v cannot have at least c connectivity because the necessary additional connectivity is greater than the number of members to be added. As a result, $c - f(v, V_{I_q}) \leq n' - 1 - |V_{I_q}|$, which can be expressed as $f(v, V_{I_q}) \geq c - n' + 1 + |V_{I_q}|$. Also, when $|V_{I_q}| \geq n'$, v needs to be connected with c other members so that the group $V_{I_q} \cup \{v\}$ can satisfy the familiarity constraint. Formally, we write the familiarity constraint function as:

$$f(v, V_{I_q}) \geq \begin{cases} c - n' + 1 + |V_{I_q}|, & \text{when } |V_{I_q}| < n' \\ c & \text{otherwise} \end{cases} \quad (5)$$

Algorithm. We can slightly modify Algorithm 1 to introduce the above familiarity constraint. The code in Line 20-28 will be executed only if checkFamiliarity(v, V_I) returns true. The procedure checkFamiliarity verifies whether v satisfies the *familiarity constraint satisfaction function* as discussed. If v fails to satisfy the constraint, v is marked as visited. Next, we explain the working procedure of our *Approximate* (which also includes all the steps of our *Exact* approach) with a running example.

5.2 Detailed Steps with an Example

We explain our algorithm with the example in Figure 1. Let $V = \{a, b, c, d, e\}$ be the set of members and $O = \{o_1, o_2\}$ be the set of meeting points. We want to find the top 2 groups, i.e., $k = 2$ where group size can vary between 3 and 4 (i.e., $n' = 3$, $n'' = 4$). Each resulting group needs to maintain minimum acquaintance constraint $c = 1$. Let $\alpha = \beta = \gamma = 0.33$. Figure 1 shows the social connectivity of the users, where the table presents the distance of the users from each meeting point in kms. Let the maximum spatial distance constraint d_m be 15kms. Initially, for each meeting point we retrieve the members within d_m distance from the R-tree in increasing order of spatial distances (Line 1.5).

Figure 2 presents the step-by-step illustration of the member exploration while forming groups. Each state (node in tree) is marked with a number denoting the sequence of exploration step. The users within d_m distance from o_1 and o_2 are $\{b, a, e, d, c\}$ and $\{c, e, a, d, b\}$, respectively, where the users are sorted in ascending order of their distances from the corresponding meeting point.

We explore the groups in a best first manner with a min-priority queue Q, where Q is maintained based on the minimum distance between the meeting point o_i and the locations of the members in V_{R_i}. Initially, there are two entries in Q: $(o_1, \{\}, \{c, e, a, d, b\}, 6)$ and $(o_2, \{\}, \{b, a, e, d, c\}, 2)$ (Refer to Section 4.5 for entries in Q). The entry $(o_2, \{\}, \{b, a, e, d, c\}, 2)$ is dequeued first from Q based on the minimum distance. Let, we begin exploring from the member b. This state of exploration is shown in Figure 2 (marked as 1). As the terminating conditions are not satisfied, according to Lines 1.25-1.26, the entries $(o_2, \{b\}, \{a, e, d, c\}, 3)$ and $(o_2, \{\}, \{a, e, d, c\}, 3)$ are then pushed to Q for further exploration. The subsequent explorations are shown in Figure 2.

From Figure 2, we see that less number of groups are explored for o_1 than o_2. For o_1, all of the subsequent states of $\{c, e\}$, $\{c, d\}$, $\{e, d\}$ and $\{c, a, d\}$ are pruned by *early termination* (Lemma 1). Similarly, the state $\{a\}$ and its subsequent states are also not generated for the same reason. State $\{e, a, d\}$ is filtered by *familiarity constraint function* (Subsection 5.1) and its subsequent states are pruned due to *early termination*. For meeting point o_2, *Exact* approach stops generating unnecessary states, i.e., state $\{d\}$ or $\{e\}$ is not generated since it cannot produce any group satisfying the minimum size constraint. *Familiarity constraint satisfaction function* stops generating some states, for example $\{b, e, c\}$, $\{e, d, c\}$, $\{a, e, d\}$. At the end of the process, group $\{b, a, d, c\}$ and $\{b, a, e, c\}$ are the top 2 groups with score 83.6 and 79.2 respectively.

6. A FAST APPROXIMATE APPROACH

In the *Exact* approach, we defined upper bounds and the terminating condition based on all the remaining set of members to ensure that all feasible groups are considered, hence the *Exact* approach needs to explore a large number of members. To expedite the group search further, we propose a *Fast Approximate (FA)* approach. The key idea is to develop *an upper bound on spatial distance and a lower bound on social connectivity* of a *member* (in contrast to all members in the exact approach) to be included in a feasible group. Based on the bounds, we early terminate when there is no remaining member who can increase the group rank. Moreover, we impose a strict familiarity constraint, i.e., we only include a member if her connectivity with the existing members of the group is greater than the *expected* connectivity of the group (cf. Section 6.4). In this approach, we only include a member to the initially formed group if it results in a higher ranked group. Let, for a meeting point o_q we have got an initial feasible group V_{I_q} where $|V_{I_q}| \geq n'$. A member $v \in V_{R_q}$ will be included in V_{I_q} if $V_{I_q} \cup \{v\}$ has a higher rank score than V_{I_q}. Similar to the exact approach (Section 4), we derive the gains in social score, spatial score, and the group size score for a new member as follows.

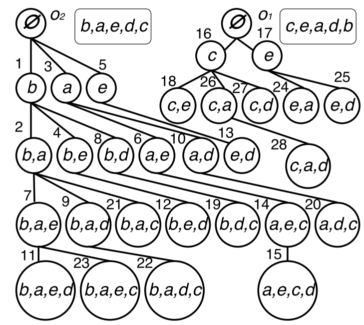


Figure 2: Node exploration steps

Social score gain: Let f_c be the total social connectivity of members in V_{I_q} and δ_f be the additional social connectivity if a *new* member v is included in V_{I_q}. Thus, the social score gain of the group $V_{I_q} \cup \{v\}$ can be expressed as follows.

$$\Delta S_{sc} = \frac{f_c + \delta_f}{(|V_{I_q}| + 1) * |V_{I_q}|} - \frac{f_c}{|V_{I_q}| * (|V_{I_q}| - 1)}$$
$$= \frac{1}{|V_{I_q}| * (|V_{I_q}| + 1)} \left(\delta_f - \frac{2f_c}{(|V_{I_q}| - 1)} \right)$$

Spatial score gain: Let d_c be the aggregate spatial distance from members in V_{I_q} to o_q, and δ_d be the additional spatial distance between new member $v \in V_{I_q}$ and o_q. The spatial score gain is:

$$\Delta S_{sp} = \left(1 - \frac{d_c + \delta_d}{d_m * (|V_{I_q}| + 1)}\right) - \left(1 - \frac{d_c}{d_m * |V_{I_q}|}\right)$$
$$= \frac{1}{d_m * (|V_{I_q}| + 1)} \left(\frac{d_c}{|V_{I_q}|} - \delta_d \right)$$

Group score gain: If we include a new user v to V_{I_q}, the group size will increase by one. Thus, the group size score gain is:

$$\Delta S_{gs} = \frac{(|V'| + 1) - |V'|}{n''} = \frac{1}{n''}$$

Total score gain: To ensure that the new group $V_{I_q} \cup \{v\}$ has a higher rank score than the initial group V_{I_q}, the summation of the gains of the above three scores must be positive.

$$\alpha * \Delta S_{sc} + \beta * \Delta S_{sp} + \gamma * \Delta S_{gs} > 0$$
$$\Rightarrow \frac{\alpha}{|V_{I_q}| * (|V_{I_q}| + 1)} \left(\delta_f - \frac{2f_c}{(|V_{I_q}| - 1)} \right) + \frac{\gamma}{n''}$$
$$+ \frac{\beta}{d_m * (|V_{I_q}| + 1)} \left(\frac{d_c}{|V_{I_q}|} - \delta_d \right) > 0$$
$$\Rightarrow \frac{d_m * (|V_{I_q}| + 1)}{\beta} \left(\frac{\alpha}{|V_{I_q}| * (|V_{I_q}| + 1)} \left(\delta_f - \frac{2f_c}{(|V_{I_q}| - 1)} \right) + \frac{\gamma}{n''} \right) + \frac{d_c}{|V_{I_q}|} > \delta_d \quad (6)$$

6.1 Distance Upper Bound for an User

Based on the above formulation we can derive a distance bound for a user v. In Eq. (6), we have two unknown variables, δ_d and δ_f. As the new group must satisfy constraint c, if we add v (from V_{R_q}) to V_{I_q}, the connectivity of group V_{I_q} will increase at least by $2 * c$. Thus, we replace δ_f with $2 * c$ in Eq. (6). Let left hand side (L.H.S) of Eq. (6) be $d^{v\uparrow}$. Thus, we get $d^{v\uparrow} > \delta_d$. Therefore, $d^{v\uparrow}$ is the *upper bound on spatial distance* of a user v to be considered as a candidate member in V_{I_q} w.r.t. o_q. Formally,

$$\frac{d_m * (|V_{I_q}|+1)}{\beta}\left(\frac{\alpha}{|V_{I_q}| * (|V_{I_q}|+1)}\left(2 * c - \frac{2f_c}{(|V_{I_q}|-1)}\right) + \frac{\gamma}{n''}\right) + \frac{d_c}{|V_{I_q}|} = d^{v\uparrow} \tag{7}$$

Here, if $d(v, o_q) < d^{v\uparrow}$, the new group $V_{I_q} \cup \{v\}$ will have higher score than that of the previous group.

LEMMA 2. *For a $v \in V_{R_q}$, if $|V_{I_q}| \geq n', d(v, o_q) < d^{v\uparrow}$,and $f(v, V_{I_q} \cup \{v\}) \geq c$, then $V_{I_q} \cup \{v\}$ guarantees a higher scoring group than the current group V_{I_q}.*

6.2 Lower Bound Social Connectivity of a User

When $v \in V_{R_q}$ satisfies Lemma 2, $V_{I_q} \cup \{v\}$ becomes higher scoring than V_{I_q}. However, if v cannot satisfy Lemma 2, $V_{I_q} \cup \{v\}$ can still have higher score as v can have more than c connection with members of V_{I_q} (as opposed to our previous assumption that v can have c social connection within V_{I_q}). We can re-write our score gain formulation, and can get the following equation for δ_f.

$$\alpha * \Delta S_{sc} + \beta * \Delta S_{sp} + \gamma * \Delta S_{gs} > 0$$
$$\Rightarrow \frac{\alpha}{|V_{I_q}| * (|V_{I_q}|+1)}\left(\delta_f - \frac{2f_c}{(|V_{I_q}|-1)}\right) + \frac{\gamma}{n''} + \frac{\beta}{d_m * (|V_{I_q}|+1)}\left(\frac{d_c}{|V_{I_q}|} - \delta_d\right) > 0$$
$$\Rightarrow \delta_f > \frac{|V_{I_q}| * (|V_{I_q}|+1)}{\alpha}\left(\frac{\beta}{d_m * (|V_{I_q}|+1)} * \left(\delta_d - \frac{d_c}{|V_{I_q}|}\right) - \frac{\gamma}{n''}\right) + \frac{2f_c}{(|V_{I_q}|-1)}$$

Here, we put $\delta_d = d(v, o_q)$, which is the lowest possible δ_d as v has the minimum distance in V_{R_q} from o_q, to get the lower bound on social connectivity $f^{v\downarrow}$ from the R.H.S of the above equation. Thus, $\delta_f > f^{v\downarrow}$. Eventually, if including v in V_I results in increased social connectivity more than $f^{v\downarrow}$, $V_{I_q} \cup \{v\}$ will have higher score than V_{I_q}. Since $f(v, V_{I_q})$ denotes the number of connectivity of v in V_{I_q}, we have $\delta_f = 2 * f(v, V_{I_q})$. Formally,

$$f^{v\downarrow} = \frac{|V_{I_q}| * (|V_{I_q}|+1)}{\alpha}\left(\frac{\beta}{d_m * (|V_{I_q}|+1)} * \left(d(v, o_q) - \frac{d_c}{|V_{I_q}|}\right) - \frac{\gamma}{n''}\right) + \frac{2f_c}{(|V_{I_q}|-1)} \tag{8}$$

LEMMA 3. *If $|V_{I_q}| \geq n'$ and $2 * f(v, V_{I_q}) > f^{v\downarrow}$, $V_{I_q} \cup \{v\}$ guarantees a higher scoring group than the current group V_{I_q}.*

PROOF. Let $d(v, o_q)$ be the minimum spatial distance between any $v \in V_{R_q}$ to o_q. We compute the lower bound on social connectivity $f^{v\downarrow}$ by putting the value $\delta_d = d(v, o_q)$ in Eq. (8). Since v has the minimum spatial distance $d(v, o_q)$ to o_q, the social connection of v in V_{I_q} must be greater than the lower bound of social connectivity to guarantee that the score of $V_{I_q} \cup \{v\}$ is greater than current V_{I_q}. As a new connection increases the social connectivity of the graph by two, if $2 * f(v, V_{I_q})$ is greater than $f^{v\downarrow}$, then $V_{I_q} \cup \{v\}$ is guaranteed to be a higher scoring group than V_{I_q}. □

6.3 Early Termination using Distance Bound

According to Lemma 2, we can decide whether we should add a member $v \in V_{R_q}$ to V_{I_q}. To terminate our search for any potential members in V_{R_q}, we have to ensure that no other members in V_{R_q} can generate any better group. However, checking such constraint for each member in V_{R_q} is computationally expensive. To overcome this, we compute an upper bound on spatial distance based on the social connectivity assumption that the member v in V_{R_q}, which is considered to be included in V_{I_q}, will be socially connected to every other members of V_{I_q}. Moreover, if $maxdeg$ denotes the maximum degree of the initial set V_{R_q} of all members, then a member cannot be connected to more than $maxdeg$ members. Thus, we get $f_{min} = min(maxdeg, |V_{I_q}|)$. We put $\delta_f = 2 * f_{min}$ in Eq. (6), and get the L.H.S. of Eq. (6) as $d^{\uparrow}$. Hence, $d^{\uparrow} > \delta_d$. Therefore $d^{\uparrow}$ is the upper bound on spatial distance for a member in V_{R_q} w.r.t. o_q. Formally, we get $d^{\uparrow}$ as follows.

$$\frac{d_m * (|V_{I_q}|+1)}{\beta}\left(\frac{\alpha}{|V_{I_q}| * (|V_{I_q}|+1)}\left(2 * f_{min} - \frac{2f_c}{(|V_{I_q}|-1)}\right) + \frac{\gamma}{n''}\right) + \frac{d_c}{|V_{I_q}|} = d^{\uparrow} \tag{9}$$

We fetch members from V_{R_q} in increasing order of their spatial distance from o_q. Let $v \in V_{R_q}$ be the member with the minimum spatial distance from o_q. Then v cannot provide any higher rank score if $d(v, o_q) > d^{\uparrow}$.

LEMMA 4. *Let $v \in V_{R_q}$ be the next fetched member from V_{R_q}. If $|V_{I_q}| \geq n', d(v, o_q) > d^{\uparrow}$, the search for a better group w.r.t. o_q can be terminated as including any member from V_{R_q} to V_{I_q} does not result in a higher scoring group.*

PROOF. Any member from V_{R_q} is expected to have spatial distance to o_q less than $d^{\uparrow}$ so that including one more member from V_{R_q} to V_{I_q} ensures positive score gain. Let v be the next retrieved member from V_{R_q}. As the members are retrieved in increasing order of their distances from o_q, v has the minimum distance from o_q than any other member in V_{R_q}. If $d(v, o_q) > d^{\uparrow}$, then no other subsequent member in V_{R_q} can have a distance less than $d^{\uparrow}$. Thus we can safely terminate as no better scoring group can be formed. □

6.4 A Heuristic for Familiarity Constraint

To expedite the group search process in the fast approximate approach, we also propose a heuristic that prioritizes members with higher social connectivity to the intermediate solution group. Since the minimum group size is n', each member in n' size group must be connected with at least c members from the remaining $n' - 1$ members to satisfy the acquaintance constraint. According to unitary method, a member $v \in V_{R_q}$ will be included in V_{I_q}, if v has social connectivity with at least $\frac{c*|V_{I_q}|}{n'-1}$ members. Thus we get the social connectivity of member v in V_{I_q}, $f(v, V_{I_q}) \geq \frac{c*|V_{I_q}|}{n'-1}$ when $|V_{I_q}| < n'$. On the other hand, when $|V_{I_q}| \geq n'$, v must know c members to form a group $V_{I_q} \cup \{v\}$ that satisfies the minimum acquaintance constraint. In summary, we can express our strict familiarity constraint satisfaction function as follows.

$$f(v, V_{I_q}) \geq \begin{cases} \frac{c*|V_{I_q}|}{n'-1}, & |V_{I_q}| < n' \\ c & |V_{I_q}| \geq n' \end{cases} \tag{10}$$

6.5 Algorithm

In the *FA* approach, we incrementally fetch members from V_R in increasing order of their distances from a meeting point, and include a member to the intermediate solution group V_I, when the member satisfies the *strict familiarity constraint function*. Once the size of V_I reaches n', we compute both spatial and social bounds to decide on whether we can include more members to the group to form higher scoring groups. The steps of the algorithm is quite similar to the *Exact* algorithm. We need to make the following changes in Algorithm 1: (i) In the `advanceTerminate` procedure (Line 19), we need to incorporate the early termination condition as prescribed in Lemma 4. (ii) The block (Line 20 - 28) is executed only when $|V_I| < n'$ is true, otherwise when either $d_v < d^{v\uparrow}$ (Lemma 2) or $2 * f(v, V_I) > f^{v\downarrow}$ (Lemma 3) is true.

6.6 Approximation Ratio

In this section, we derive a theoretical bound on the approximation ratio of our approximate approach. We compute the ratio as the score of a group retrieved by our *Fast Approximation* algorithm divided by the score of the best possible group which might be missed by our algorithm in the worst case scenario.

In our approach, the set of unexplored members, V_R is sorted according to distance of the members from a meeting point. Let G be a group retrieved from V_R where d_L and d_H are the nearest and the farthest distances of members from the meeting point o, respectively. As per Equation 1, the score of group G w.r.t. o, $S(G, o)$ (or simply $S(G)$), will be the lowest when each member in G has a social connectivity of exactly c (the lowest connectivity) and distance d_H from o. Let us denote such lowest scoring group (sub-optimal) as G_{sopt}. Similarly, the highest scoring group is formed when each member is connected to every other member in the group and has a distance of d_L from the meeting point. Let us denote this group (optimal) as G_{opt}. Let us assume that the sizes of G_{sopt} and G_{opt} are n and n_{opt}, respectively. Since we assume that members of G_{opt} are at d_L distance, and G_{sopt} are at d_H distance, and our algorithm retrieves members in order of distance, we can retrieve members of G_{sopt} instead of members of G_{opt} iff $d_H = d_L$. We compute the score of each of the three constituents of our G_{sopt} and G_{opt} as follows.

Score	G_{sopt}	G_{opt}
Social	$\frac{2\times n\times c/2}{n(n-1)} = \frac{c}{n-1}$	$\frac{2\times n_{opt}\times(n_{opt}-1)/2}{n_{opt}(n_{opt}-1)} = 1$
Spatial	$1 - \frac{n\times d}{n\times d_m} = 1 - \frac{d}{d_m}$	$1 - \frac{n_{opt}\times d}{n_{opt}\times d_m} = 1 - \frac{d}{d_m}$
Size	$\frac{n}{n''}$	$\frac{n_{opt}}{n''}$

Hence, we have the following scores: $S(G_{sopt}) = \alpha \times \frac{c}{n-1} + \beta \times (1 - \frac{d}{d_m}) + \gamma \times \frac{n}{n''}$, and $S(G_{opt}) = \alpha \times 1 + \beta \times (1 - \frac{d}{d_m}) + \gamma \times \frac{n_{opt}}{n''}$

If we set $n_{opt} = n''$ (the maximum group size), we get the maximum possible score of $S(G_{opt})$ as: $S(G_{opt}) = \alpha \times 1 + \beta \times (1 - \frac{d}{d_m}) + \gamma \times 1$. Hence, for any group returned by the *FA* algorithm, the approximation ratio will be bounded by the following value:

$$\frac{S(G_{sopt})}{S(G_{opt})} = \frac{\alpha \times \frac{c}{n-1} + \beta \times (1 - \frac{d}{d_m}) + \gamma \times \frac{n}{n''}}{\alpha \times 1 + \beta \times (1 - \frac{d}{d_m}) + \gamma \times 1}$$

We also show the approx. ratio bound for different scenarios:

Emphasis	Weights	Approximation ratio
Social score	$\alpha = 1, \beta = \gamma = 0$	$\frac{c}{n''-1}$
Spatial score	$\beta = 1, \alpha = \gamma = 0$	1
Size score	$\gamma = 1, \alpha = \beta = 0$	$\frac{n'}{n''}$

6.7 A Greedy Approximation Approach

To expedite the search, we propose a greedy approximate approach that works as our baseline for the approximate approach. In the greedy approximation approach, we avoid the backtracking, and only progressively add members from V_R to V_I if they satisfy familiarity and other constraints. Thus, when a member is included in V_I we do not exclude it for forming other possible groups.

7. EXPERIMENTAL EVALUATION

In this section we present the experimental evaluation for the baseline and our proposed approaches to answer the *Top k-FSSGQ* queries. Specifically, we compare the performance among the following five methods: (i) the baseline (*B*) as presented in Section 3, (ii) the exact approach (*E*) (iii) the *approximate* approach *(A)*, (iv) the *fast approximate* approach (*FA*), and (v) the *greedy approximate* approach (*GA*) as presented in Section 6.7.

7.1 Experimental Settings

These algorithms are implemented in Java and run on a server with Intel Xeon E5-2630, 6 cores X 2 threads per core @2.3 Ghz, 15360 kB of cache and 256 GB of RAM.

Dataset. We conduct extensive experiments with three real datasets (i) Brightkite [17], (ii) Gowalla [17], and (iii) Twitter [1].

Brightkite and Gowalla, each contains the social connections of the users and their check-in locations. As a user may have multiple check-ins, we consider the most frequent check-in location as the location of that user. If a user does not have any check-in, that user along with her social connections is discarded. The meeting point locations are generated by using the same distributions of check-in locations, i.e., the locations with higher check-ins have higher chance of selecting as meeting locations. Table 2 contains the details information of the datasets after applying these processing.

Table 2: Datasets

Datasets	#Nodes	#Edges	Check-ins	Time Period
Brightkite	58,228	214,078	4,491,143	Apr 08 - Oct 10
Gowalla	196,586	950,327	6,442,890	Feb 09 - Oct 10
Twitter	10M	84,744,091	-	May 11

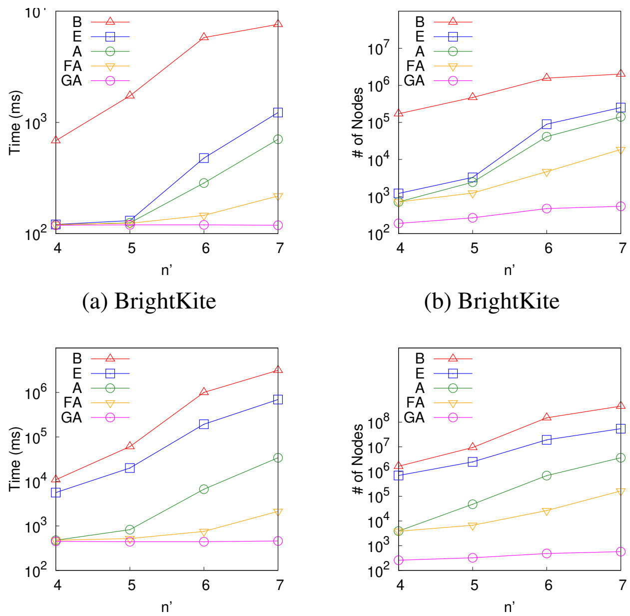
The intuition behind selecting the meeting points in this way is, if many people frequently visit a place, the place is more likely to be their meeting point in real life. We also ran experiments by randomly selecting the meeting points with uniform distribution. However, in that case, there are many instances where no valid group is formed due to the sparsity of check-ins. Therefore, we exclude such experiments results.

The Twitter dataset contains the 'follow' relations among users and the locations of users in their profiles. As only a fraction of the users (appx. 1.5 millions of 10 millions) have their meaningful location mentioned in the profile, we generate the locations of the other users following the same distribution. We consider the 'follow' relationship as an undirected social connection. We use this augmented twitter dataset to show the scalability of our approaches.

Evaluation Metrics and Parameters. We evaluate the efficiency, scalability, and effectiveness of our algorithms by varying different parameters. The list of parameters with their ranges and default values are shown in Table 3. For all experiments, a single parameter is varied while keeping the rest at default. To determine efficiency and scalability, we study (i) total runtime and (ii) total number of

Table 3: Parameters

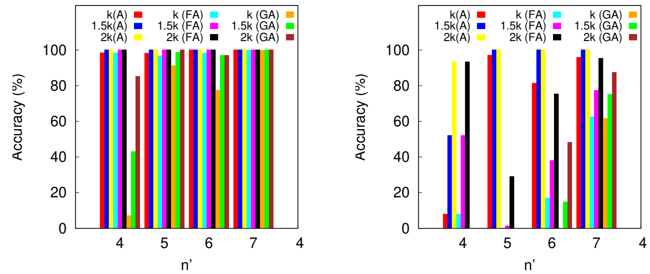
Parameter	Range	Default
Min group size (n')	4,5,6,7	6
Max group size (n'')	6,7,8,9	8
Min acquaintance	2,3,4,5	3
Max distance constraint	16,20,24,28	20
α, β, γ	[0, 1]	.33
No. of meeting points	50,100,200,400	100
k	4,8,16,32	8



(a) BrightKite (b) BrightKite

(c) Gowalla (d) Gowalla

Figure 3: Effect on varying n'



(a) Brightkite (b) Gowalla

Figure 4: Effectiveness for varying n'

members (nodes) explored to find the top-k results. For each experiment, we generate 100 queries with the same parameter setting and report the average performance. To measure the effectiveness of our approximate approaches, the impact of each parameter is studied using the following metrics.

(i) Percentage of group appearance. Both the exact and approximate approaches return a ranked list, where the groups in the approximate result is guaranteed to have a lower or equal score than the groups returned by the exact approach. Therefore, some groups in the exact results may not appear in the approximate results. In our experiments, we compute the number of groups that are common in the top k approximate results and in the corresponding top k, $1.5 * k$, and $2 * k$ exact results as a percentage. For example, when $k = 16$, we compute the percentage of the groups in top 16-approximate that also appear in top 16, top 24, and top 32-exact. Similar evaluation is applied for top k-*FA* (fast approximate) and top k-*GA* (greedy) solution.

(ii) Precision. Precision is generally measured as the fraction of the relevant instances among the retrieved instances. In our case, we measure the precision as how many groups in top k-*A* are in top k-*E*. For example, when $k = 16$, if 12 groups in top 16-*A* also appear in top 16-*E*, the precision is computed as 12/16=0.75.

(iii) Recall. Recall is generally measured as the fraction of the relevant instances that have been retrieved over the total amount of relevant instances. We measure the recall as ratio of the k^{th} group rank in top k-*A* and the rank of the same group using the *E* approach. For example, when $k = 16$, if 16^{th} group in top 16-*A* appear as the 20^{th} ranked group in E, recall is 16/24=0.67.

(iv) Percentage of user overlap. We also compare the percentage of user overlaps in two rank lists. For example, when $k = 16$, if the same set of users appear in both top 16-*A* and top 16-*E*, then percentage of user overlap will be 100%.

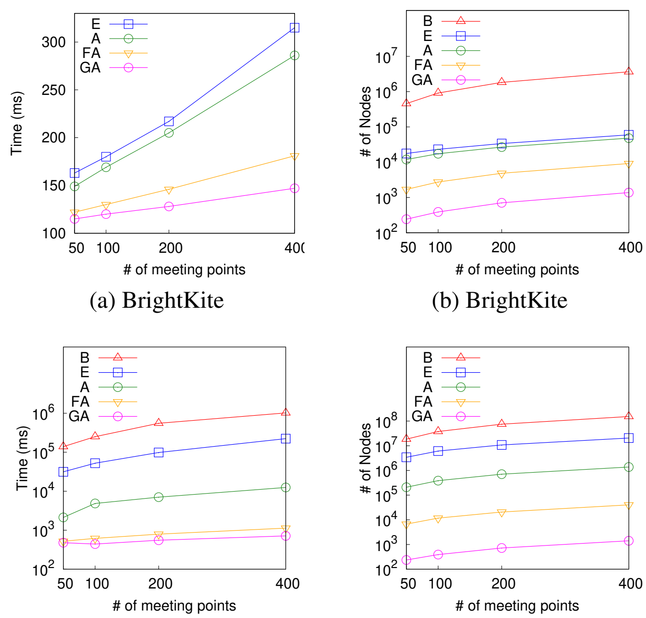
7.2 Performance evaluation

7.2.1 Varying Minimum Group Size, n'

Efficiency and scalability evaluation: Figure 3 shows the effect of varying n' for Brightkite and Gowalla. The runtime of the baseline, the exact approach, and the approximate approach gradually increase with n', as more groups are likely to be explored for a higher n'. The runtime of *FA* and *GA* does not vary much, as these processes can terminate earlier by exploring less number of groups. As shown in Figure 3, on average *E* takes 3.67 times and 9.3 times less run time than *B* in *Gowalla* and *Brightkite*, respectively. The approximate approach runs 84 times and 12.67 times (on average) faster than *B* in *Gowalla* and *Brightkite*, respectively. *FA* runs 737 times and 23.56 times (on average) faster than *B* in *Gowalla* and *Brightkite*, respectively. The number of nodes explored also shows similar trend. As *GA* avoids backtracking, the approach is faster than the other approximate approaches. Since the density of members around meeting points is higher in *Gowalla* than in *Brightkite*, the number of explored nodes w.r.t. the same group size is larger in *Gowalla* than that of *Brightkite*.

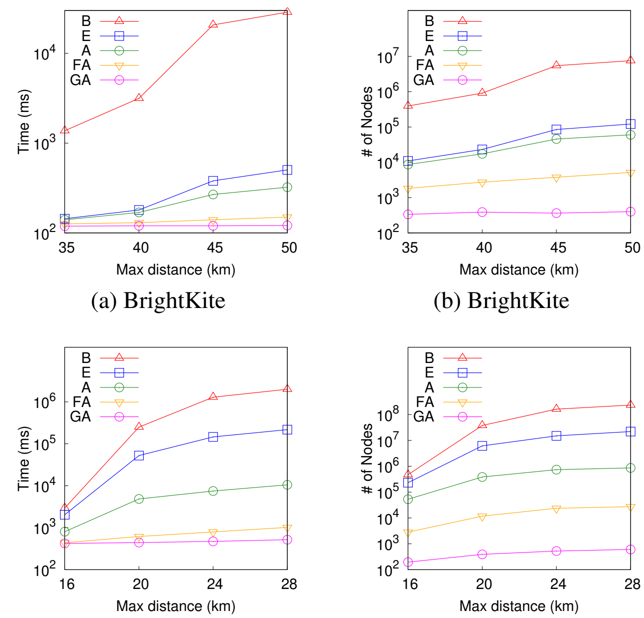
Effectiveness Evaluation: Figure 4 presents the effectiveness for varying n'. For each n', we have nine values (shown with bars). The first three values denote the percentage of the groups of top k-Approximate (*topk-A*) appearing in top k-Exact (*topk-E*), top $1.5 *$ k-*E*, and top $2 * k$-*E* rank list, respectively, where k is set to default. The next six bars represent these values for *FA* (fast approximate) and *GA* (greedy). A higher percentage denotes a higher effectiveness. For *Brightkite*, the percentage for *(A)* is 98.25-100%, and for *Gowalla* the percentage varies from 80-97% with some noticeable less percentage (8%) for lower n'. We observe that *GA* fails to find top groups of size 4 where strict familiarity is applied ($c = 3$) for *Gowalla*. However we observe that in most of the cases, the top k-*E* and the top k-*A* return exactly the same set of groups. *FA* produces similar result (96.75-99.75%) for *Brightkite*, whereas 63% in the average best case for *Gowalla* dataset.

High precision (99% for *Brightkite* and 70% for *Gowalla*) and high recall are also observed for approximation. The precision of *(FA)* is on average 98% and 21% for *Brightkite* and *Gowalla*, respectively. However recall of *FA* is 98% and 46% for these datasets.



(a) BrightKite (b) BrightKite (c) Gowalla (d) Gowalla

Figure 5: Effect on varying number of meeting points



(a) BrightKite (b) BrightKite (c) Gowalla (d) Gowalla

Figure 6: Effect on varying maximum distance

The percentage of user overlaps between the top k-*E* and the top k-*A* is always 100% for *Brightkite*, whereas, the percentage user overlaps is on average 81% for *Gowalla* (not shown in figure). Similar result of member overlap is found between the top k-*E* and the top k-*FA* (always 100% for *Brightkite* and 50.65% for *Gowalla*).

GA (greedy) exhibits lower percentage of group appearance for *Gowalla*. Sometimes, *GA* has a very low percentage (e.g., 0% for *Gowalla*). *GA* exhibits 69% group appearance in top k-*E*, 85% in top $1.5 * k$-*E*, and 96% in $2 * k$-*E* for *Brightkite*. In general, both precision and recall (79% for *Brightkite* and 31% for *Gowalla*) are also much lower (not shown in figure) than the other approximate approaches. Although *GA* has much lower runtime and the least number of node exploration, the effectiveness is traded significantly for efficiency in many cases.

7.2.2 Varying Meeting Points

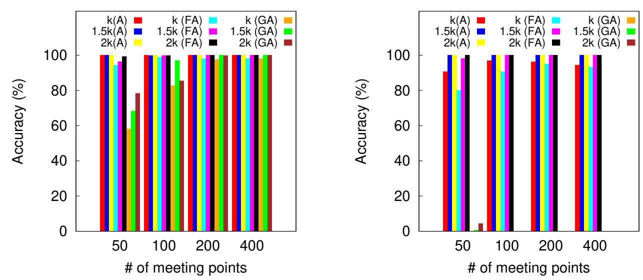
Efficiency and scalability: As the search space increases with the increase of the $|O|$, the costs for the baseline and exact approach increase (Figure 5). The costs of the approximate algorithms do not vary much as many calculations are avoided by considering bounds on individual members than the group. The benefit of our approaches are higher for higher $|O|$. Similar pattern is seen w.r.t. number of nodes explored.

Effectiveness: Figure 7 shows the percentage of group appearance when we vary $|O|$. Both *A* and *FA* demonstrate a very high effectiveness for both datasets. *GA* has a very low percentage (0%) for *Gowalla* but high percentage (84%) for *Brightkite*. *GA* produces similar results while varying other parameters too.

From these experiments we consistently find that, although *GA* has almost constant efficiency, the effectiveness of *GA* is not competitive in many cases.

7.2.3 Varying Maximum Distance Threshold (d_m)

As more users become eligible to be included in the result with the increase of d_m, costs increase rapidly with the increase in d_m



(a) Brightkite (b) Gowalla

Figure 7: Effectiveness for varying the number of meeting points

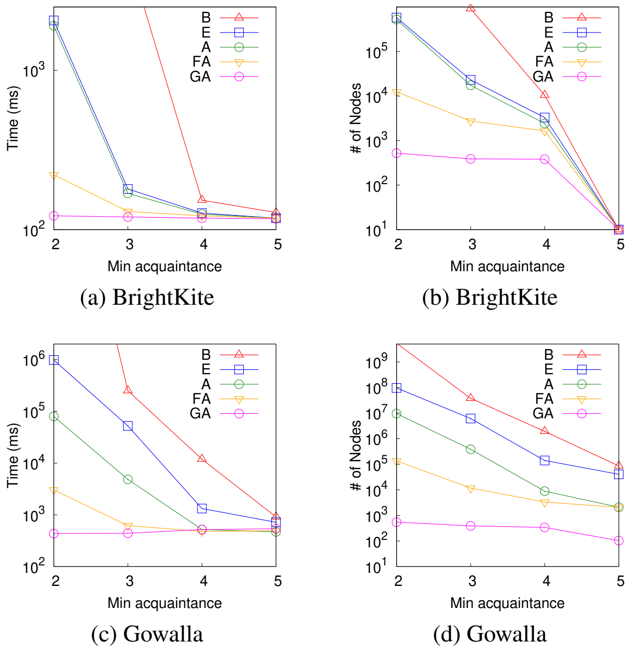
(Figure 6). In all cases, baseline is significantly outperformed by other approaches. *E* takes 13 times higher runtime than *A* and 122 times higher runtime than *FA* for threshold 50km in *Gowalla*. The number of nodes explored also increases for a higher threshold due to the expanded search space. As the density of members is higher in *Gowalla* than *Brightkite*, the benefit of the both *A* and *FA* is much higher than *E* in *Gowalla* for a higher threshold.

7.2.4 Varying Minimal Acquaintance Constraint c

Figure 8 demonstrates that for both datasets, the number of explored nodes decreases rapidly with the increase of c. The reason is that, the degree of nodes in all datasets follow the *long tail* distribution. So for a higher c, the number of nodes that do not satisfy the constraint increases rapidly; thus a higher number of nodes can be pruned. Although the number of nodes explored differ in all cases, the runtimes are very close for $c \geq 4$ for *Brightkite* and *Gowalla*.

7.2.5 Varying Other Parameters

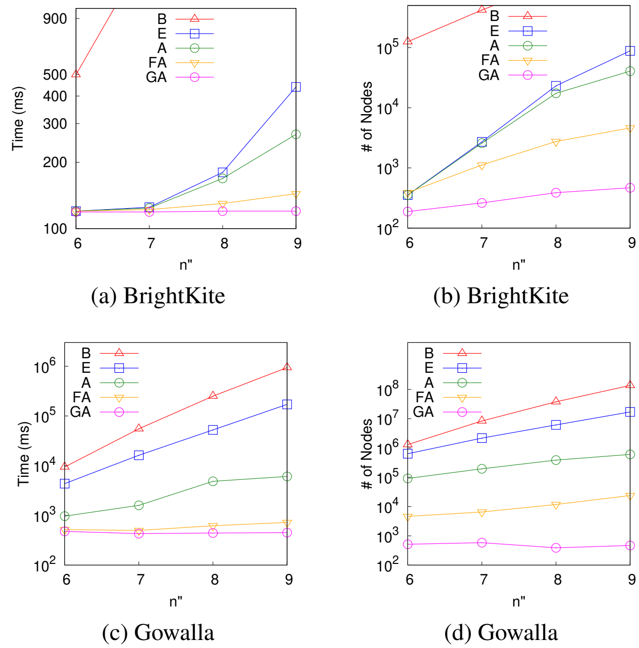
We also vary k, α, β, and γ (not shown for space constraint). We do not observe any noticeable change in trends for varying these parameters except β. With the increase of β, especially when $\beta > \alpha$



(a) BrightKite (b) BrightKite

(c) Gowalla (d) Gowalla

Figure 8: Effect on varying minimum acquaintance constraint



(a) BrightKite (b) BrightKite

(c) Gowalla (d) Gowalla

Figure 9: Effect on varying maximum group size

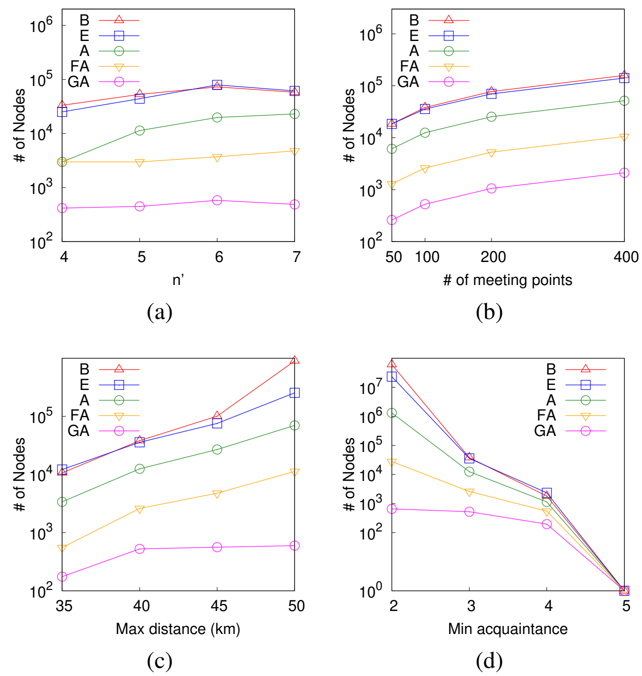
and $\beta > \gamma$, both the processing time and the number of nodes explored decrease. Since we explore members based on the increasing order of spatial distance w.r.t. meeting points, it is expected that the nearby groups are found quickly for a large β.

7.2.6 *Experiments with Twitter Dataset*

To show our scalability, we use augmented Twitter dataset of 10 million users, and run the experiments by varying different parameters (Figure 10(a) - (d)). Due to space constraints, we have only shown the number of nodes explored (which shows similar trends to the required time). We have observed similar performance improvement that we observed in other datasets. However, we have also observed few exceptions: e.g., in Figure 10(a), the results do not change much for varying n', the number of nodes (also the time) even start to decrease after $n' = 6$. The reason is that, on average, each user has only a few connections in this dataset. Thus, if n' is fixed to a large value, not many groups can satisfy that constraint. However, *A*, *FA*, and *GA* require about 5.5, 15, and 110 times less node exploration than the baseline.

8. CONCLUSIONS

We have proposed a novel *Top k Flexible Socio Spatial Group Query* (*Top k-FSSGQ*) to find the top k groups of various sizes w.r.t. multiple POIs. To incorporate the trade-offs among different socio-economic factors, we have devised a ranking function by combining social closeness, spatial distance, and group size, which provides the flexibility of choosing groups of different sizes under different constraints. To effectively process *Top k-FSSGQ*, we have first developed an *Exact* approach that ensures early termination of the search based on the computed upper bound distance. We have proved that the problem is NP-hard, and thus we have designed a *Fast Approximate* approach based on the relaxed bound and strict social connectivity constraint, which is much faster than the exact solution by sacrificing the quality slightly. We have conducted detailed experimental studies with three popular real-world datasets



(a) (b)

(c) (d)

Figure 10: Scalability experiments on the Twitter dataset

and shown that the *Exact* approach runs up to one order magnitude faster than the baseline, and the *Fast Approximate* approach runs up to two orders of magnitude faster than the *Exact* approach and returns the same set of groups in most of the cases.

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