Efficient Regular Simple Path Queries under Transitive Restricted Expressions

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ABSTRACT

There are two fundamental problems in regular simple path queries (RSPQs). One is the reachability problem which asks whether there exists a simple path between the source and the target vertex matching the given regular expression, and the other is the enumeration problem which aims to find all the matched simple paths. As an important computing component of graph databases, RSPQs are supported in many graph database query languages such as PGQL and openCypher. However, answering RSPQs is known to be NP-hard, making it challenging to design scalable solutions to support a wide range of expressions. In this paper, we first introduce the class of *transitive restricted expression*, which covers more than 99% of real-world queries. Then, we propose an efficient algorithm framework to support both reachability and enumeration problems under transitive restricted expression constraints. To boost the performance, we develop novel techniques for reachability detection, the search of candidate vertices, and the reduction of redundant path computation. Extensive experiments demonstrate that our exact method can achieve comparable efficiency to the state-of-the-art approximate approach, and outperforms the state-of-the-art exact methods by up to 2 orders of magnitude.

KEYWORDS

Regular expression, Simple path, Reachability, Enumeration

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1 INTRODUCTION

Graphs are often used to model entities and their connections in complex systems, such as social networks, the World Wide Web, and computer networks. In many real applications, each edge between two vertices is associated with a label to represent a specific relation. In recent years, *regular path queries* have been extensively studied on edge-label graphs [1, 3, 11, 31]. There are two fundamental problems: (1) the *reachability query* asks whether there exists a path between a given pair of source and target vertices satisfying the regular expression [10, 19, 32, 41], and (2) the *enumeration query* aims to find all paths between two given vertices that satisfy a given regular expression [12, 26, 27]. In the literature, regular path queries are usually studied under some possible semantics, including arbitrary path, shortest path, and simple path [27]. To avoid overwhelming and redundant results, in this paper, we focus on regular path queries under simple path semantics, named *Regular Simple Path Queries (RSPQs)*, which requires that there are no repeated vertices along a resulting path.

Applications. RSPQs can be applied in many real scenarios. Some examples are the following.

(1) Knowledge Retrieval in Knowledge Graphs. RSPQs are building blocks in many graph query languages such as PGQL [40] and openCypher[1] [15, 34]. In knowledge graphs or graph databases, many information retrieval tasks can be solved by RSPQs. For example, if we want to know whether a user U lives in New York City in a knowledge graph, we should check whether there exists a simple path from user U to New York City. In addition, the label of the path should be limited since we only consider the relationship of living rather than other relationships. We can use the Reachability Query $(U, NewYork, (LivedIn \circ (PartOf)^*)$ to answer the above question where $LivedIn$ records the place of residence, and $PartOf$ denotes the containment relationship of territorial entities. We use the regular expression $(PartOf)^*$ to expand the place of residence to the city level, because the residence may be recorded as a street rather than a city. If we can find a simple path satisfying the expression such as $U \xrightarrow{LiveIn}$ Grand Street $\xrightarrow{PartOf}$ New York, then we know U lives in New York City.

(2) Cyber-attack Detection in Computer Network Traffic. In the cybersecurity community, advanced persistent threat (APT) detection has been one of the most important tasks. Recent advances in APT

[1]The paths supported in openCypher are restricted to simple paths (as described in Section 2.3 of [34]), Similarly, PGQL also supports regular simple path queries.

Table 1: Structure of property paths from Table 4 in [8]. A **means** $(a_1 + a_2 \cdots + a_k)$.

Name	Relative	LCR?	TRE?	Name	Relative	LCR?	TRE?
a^*	50.48%	✓	✓	A^*	0.60%	✓	✓
$a_1 \circ a_2 \cdots \circ a_k$	24.26%	×	✓	$a \circ b^* \circ c$	0.22%	×	✓
$a \circ b^*$	17.07%	×	✓	$a^* \circ b^*$	0.11%	×	✓
A	5.52%	×	✓	$\epsilon \mid A$	0.06%	×	✓
$a \circ b^* \circ c^*$	1.49%	×	✓	$a \circ b \circ c^*$	0.05%	×	✓

detection tend to utilize the provenance graph [2, 17, 29], where vertices are network entities, and edges represent activities. An APT attack usually consists of a sequence of network activities, which can be easily detected by a pattern-matching method. However, in the real world, the attackers may add noise to their activities to escape detection [29]. As described in [29], in the provenance graph, a compromised browser writing to a system file may correspond to a path where a vertex representing a Firefox process forks new processes, only one of which ultimately writes to the system file. By modeling such paths as RSPQs, e.g., $(Browser, File, ((Fork)^* \circ Write))$, we are able to hunt suspicious APTs.

(3) Pathway Analysis in Biological Networks. Pathway queries are essential in the analysis of biological networks, where vertices represent entities such as compounds and edges represent various forms of interactions [13, 19, 24]. For instance, in [19], a network analyzer can quickly determine if two compounds have a given pathway, which refers to specific forms of interactions, using reachability queries. Additionally, simple path enumeration is a common type of pathway query in biological networks [13], which provides detailed information about the paths between entities.

Limitations of Existing Studies. Although RSPQs have been extensively studied [1, 5, 9, 11, 27, 28, 41], there are several major limitations in existing solutions to RSPQs.

(1) Only support a small subset of regular expressions. Most existing works [32, 39, 45] only focus on label-constraint reachability (LCR) queries. Given a set of labels $L = \{a_1, \cdots, a_k\}$, an LCR query requires the label of each edge on the resulting path must belong to L. This type of label constraint could be expressed to the regular expression $(a_1 + \cdots + a_k)^*$. A recent analysis of SPARQL query logs [8] suggests that $\approx$ 48% of the RSPQs cannot be expressed by LCR .

(2) Unable to balance the efficiency and accuracy. The reachability query of RSPQs is NP-hard under arbitrary expressions [28]. Therefore, exact methods that support all regular expressions, such as BBFS [41], are computationally expensive, and even unacceptable in some extreme cases. Wadhwa et al. [41] propose ARRIVAL, an approximate and efficient method to solve this problem, but its accuracy relies on the graph structure, limiting its applicability.

(3) No efficient method for enumeration problem of RSPQs. All the above works only study the reachability problem. To the best of our knowledge, recent research [27] has proposed a polynomial delay algorithm (nearly $O(n^3)$ delay, where n means the number of vertices in the graph) for the enumeration problem of RSPQs, which only supports a subset of regular expressions called downward closed expressions. However, it is still not efficient enough and does not support more complex expressions.

Challenges. Because the reachability query of RSPQs is a well-known NP-hard problem, it is impossible to design an exact method to solve RSPQs efficiently under arbitrary expressions unless P = NP. Additionally, even though index structures can greatly enhance computational efficiency, it becomes impractical to construct a suitable index as the diversity of expression types increases. Moreover, constructing an index incurs huge time and space costs, which are unacceptable for large graphs with a large number of labels. Therefore, on the one hand, approximate and index-free algorithms for reachability queries is a feasible solution, which has been studied in [41]. On the other hand, accurate answers hold better significance in various real scenarios, motivating us to design exact methods rather than approximate ones.

The above limitations and challenges motivate us to ask the following questions. Is there a specific type of regular expression that can cover the most frequently encountered expressions in real-world scenarios, and can we develop efficient index-free algorithms to address the above two problems for such special regular expressions? In this paper, we aim to propose an efficient, exact, and index-free approach to solve the above two issues of RSPQs.

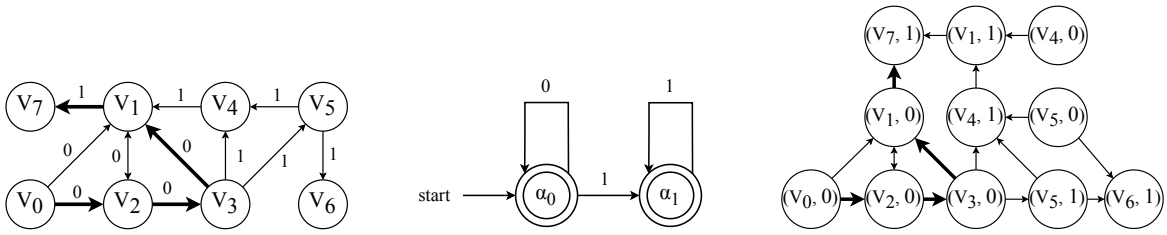
Contributions. We observe that there are two basic categories of regular expressions for which it is possible to devise efficient and exact algorithms. Based on this observation, we define a regular expression framework, called *transitive restricted expression* (TRE), which is able to cover more than 99% of regular expressions encountered in real-world scenarios from two studies [7] and [8]. Specifically, TRE could cover all the expression types (more than 99%) in Table 1 while LCR only supports two types of them (around 51.08%). Similar results can be found in [7].

To efficiently handle the TRE, we propose efficient and effective query techniques. In specific, we first divide a regular expression into three parts Pre, $Type$, and Suf. Then, we propose an approach for the two issues of RSPQs by dealing with the three parts in two phases. In the first phase, we develop an algorithm to find all the simple paths that match Pre and Suf respectively. In the second phase, we process $Type$ in addition to the output of the first phase. By combining the results of the above two phases, we obtain the final answers. Our principal contributions are the following.

- We propose a new type of regular expressions named TRE consisting of three parts Pre, $Type$, and Suf which can cover more than 99% of expressions in real-world queries.
- Based on TRE constraints, we develop an index-free and exact framework, which can solve both reachability and enumeration queries efficiently.
- To further improve the performance, we develop novel techniques for reachability detection, the search of candidate vertices, and the reduction of redundant path computation.
- Our empirical study shows that our approach can return exact solutions while achieving comparable efficiency to the state-of-the-art approximate method. It is also demonstrated that our approach outperforms the state-of-the-art exact approaches by up to two orders of magnitude in terms of efficiency.

2 PRELIMINARIES

In this section, we first formally introduce the definition of directed labeled graphs. Then we give the problem statement of regular simple path reachability and enumeration query.



(a) Edge-labeled graph G (b) DFA A (c) Product graph $P_{G,A}$

Figure 1: 1(a) A edge-labeled graph G, 1(b) the automation for expression $0^* \circ 1^*$, 1(c) the product graph $P_{G,A}$ based on G and A.

A directed labeled graph is denoted as $G = (V, E, \mathcal{L}, \phi)$, where V is a set of vertices, $E \subseteq V \times V$ is a set of directed edges, $\mathcal{L}$ is a finite non-empty set of labels, and $\phi : E \rightarrow \mathcal{L}$ is a function that maps every edge to a label. Note that multiple edges between two vertices must have distinct labels. Given two vertices $s, t \in V$, a path p from s to t in G is a sequence of edges : $\langle (v_0, l_0, v_1), \cdots, (v_{n-1}, l_{n-1}, v_n) \rangle$ where $v_0 = s$, $v_n = t$ and $l_i \in \mathcal{L}, 0 \leq i \leq n-1$. If there is no repeating vertex in path p, we say p is a simple path. The label of a path p is denoted by $\phi(p) = l_0 l_1 \cdots l_{n-1} \in \mathcal{L}^*$.

Definition 1 (Regular Expression). *A regular expression R over the alphabet $\mathcal{L}$ is defined as $R ::= \epsilon \mid l \mid R_1 \circ R_2 \mid R_1 + R_2 \mid R^*$, where (i) ϵ denotes the empty string, (ii) $l \in \mathcal{L}$ denotes a character in the alphabet, (iii) $\circ$ denotes the concatenation operator, (iv) $+$ denotes the alternation operator, (v) R, R_1 and R_2 are regular expressions, and (vi) $*$ represents the Kleene star. A regular language $L(R)$ is the set of strings that can be described by regular expression R. We say that a path p matches a regular expression R if the label of p could be described by R, i.e., $\phi(p) \in L(R)$.*

Example 1. *In Figure 1(a), considering the regular expression $R = 0^* \circ 1^*$, the path $V_0 \xrightarrow{0} V_2 \xrightarrow{0} V_3 \xrightarrow{0} V_1 \xrightarrow{1} V_7$ (indicated by the bold edges) corresponds to a match for the expression R because the label of path p, which is 0001, can be described by $0^* \circ 1^*$.*

Next, we introduce a type of restricted regular expression called *transitive restricted expression*. TRE is a combination of length-fixed expression (LE) and downward closed expression (DCE). Hence, before introducing TRE, we first give the definition of LE and DCE.

Definition 2 (Length-fixed Expression). *A length-fixed expression R is the expression that does not contain the Kleene star, i.e., $R ::= \epsilon \mid l \mid R_1 \circ R_2 \mid R_1 + R_2$, where R_1 and R_2 are LEs.*

Definition 3 (Downward Closed Expression [27]). *An expression R is called downward closed if, for any sequence $L = l_1 l_2 \cdots l_n$ that can be described by the expressions R, any subsequence $l_{i_1} \cdots l_{i_k}$, $0 < i_1 < \cdots < i_k < n+1$ can also be described by R.*

Remark 1. *$R = (l_1 + l_2 + \cdots + l_k)^*$, where $l_i \in \mathcal{L}, k \in \mathbb{N}$, represents a fundamental type of downward closed expressions. Moreover, the expression $R = R_0 \circ R_1 \circ \cdots \circ R_k, k \in \mathbb{N}$ is also downward closed if $\forall i, 0 \leq i \leq k, R_i$ is downward closed.*

Definition 4 (Transitive Restricted Expression). *We define the transitive restricted expression as $Pre \circ Type \circ Suf$, where Pre and Suf are LEs, and $Type$ is a DCE.*

Table 2: Frequently used notations.

Notations	Definitions
$A.\alpha_0$	the start state of DFA A
$A.F$	the final state set of DFA A
$N^P_{out}(u, \alpha)$	the out-neighbors of vertex (u, α)
$N^P_{in}(v, \beta)$	the in-neighbors of vertex (v, β)
$P_{G,A}$	the product graph $P_{G,A}$

With the formalization of the above concepts, now we establish the definition of regular simple path reachability query and enumeration query respectively.

Definition 5 (Regular Simple Path Reachability Query). *Given a graph G, a transitive restricted expression R, the source vertex s and the target vertex t, the regular simple path reachability query (Reachability Query) returns true if there exists at least one simple path between s and t that matches the expression R.*

Definition 6 (Regular Simple Path Enumeration Query). *Given a graph G, a transitive restricted expression R, the source vertex s and the target vertex t, the regular simple path enumeration query (Enumeration Query) returns all simple paths $P = \{p \mid p$ is a simple path and $\phi(p) \in L(R)\}$ between s and t.*

Example 2. *In Figure 1(a), assuming the source vertex is V_0, the target vertex is V_6 and the transitive restricted expression R is $0^* \circ 1^*$. The Reachability Query will return true while the Enumeration Query will give two simple paths that matched the expression R as $V_0 \xrightarrow{0} V_2 \xrightarrow{0} V_3 \xrightarrow{1} V_5 \xrightarrow{1} V_6$ and $V_0 \xrightarrow{0} V_1 \xrightarrow{0} V_2 \xrightarrow{0} V_3 \xrightarrow{1} V_5 \xrightarrow{1} V_6$.*

3 EXISTING SOLUTION

Given Reachability Query and Enumeration Query in a directed labeled graph G, we use different algorithms for these two queries generally. For both of them, we usually use deterministic finite automaton and product graphs to solve these problems. Therefore, we provide the definition of deterministic finite automaton and product graph. We introduce a basic DFS algorithm using a product graph. This algorithm is a part of our optimal solutions. Then, we introduce Bidirectional BFS and DFS-based algorithms in the product graph for these two kinds of queries respectively.

Definition 7 (Deterministic Finite Automaton). *Given a regular expression R, its corresponding deterministic finite automaton*

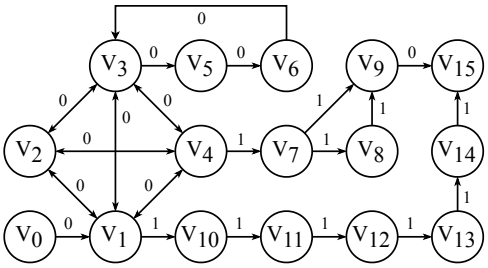


Figure 2: A bad-case example for RSPQs.

(DFA) is defined as $A = (S, \mathcal{L}, \delta, \alpha_0, F)$, where S is a set of states, $\mathcal{L}$ is the input alphabet, $\delta : S \times \mathcal{L} \rightarrow S$ is the state transition function, $\alpha_0 \in S$ is the start state of DFA, and $F \subseteq S$ is the set of final states of DFA. δ^* is the extended transition function defined as $\delta^*(\alpha, w \circ l) = \delta(\delta^*(\alpha, w), l)$, where $\alpha \in S, l \in \mathcal{L}, w \in \mathcal{L}^*, \delta^*(\alpha, \epsilon) = \alpha$, and ϵ is the empty string. We call a string w can be accepted by A if $\delta^*(\alpha_0, w) = \alpha_f, \alpha_f \in F$. Note that if w can be described by R, it must be accepted by the corresponding DFA A.

Definition 8 (Product Graph). *Given a graph $G = (V, E, \mathcal{L}, \phi)$ and a DFA $A = (S, \mathcal{L}, \delta, \alpha_0, F)$, the corresponding product graph $P_{G,A}$ is defined as $P_{G,A} = (V_P, E_P)$, where $V_P = V \times S$, $E_P \subseteq V_P \times V_P$, $((u, \alpha), (v, \beta)) \in E_P$ iff $(u, v) \in E$ and $\delta(\alpha, \phi(u, v)) = \beta$. $N^P_{out}(u, \alpha) = \{(v, \beta) \mid ((u, \alpha), (v, \beta)) \in E_P\}$ denotes the out-neighbors of vertex (u, α) and $N^P_{in}(v, \beta) = \{(u, \alpha) \mid ((u, \alpha), (v, \beta)) \in E_P\}$ denotes the in-neighbors of vertex (v, β).*

Example 3. *Figure 1(b) shows the DFA A of regex $0^* \circ 1^*$, and Figure 1(c) shows the product graph $P_{G,A}$.*

3.1 DFS based algorithm

Note that every path in G has its unique corresponding path in $P_{G,A}$. Meanwhile, for any path p' in $P_{G,A}$, G has its unique corresponding path. Hence, the intuitive idea to find a path matching R is finding the corresponding path in $P_{G,A}$.

We present a fundamental DFS algorithm to handle both Reachability Query and Enumeration Query. For a given graph G, regex R, and start vertex s, we construct the corresponding DFA A and product graph $P_{G,A}$. The DFS algorithm starts from the vertex $v_s = (s, A.\alpha_0) \in V_p$. v_s presents a vertex in $s \in V$ and the state $\alpha_0 \in S$. Then we search from v_s. If there exists an eligible simple path from source vertex s with start state to target vertex t with final states in $P_{G,A}$, we conclude that Reachability Query is true. If we identify all potential paths and none of them is eligible, the answer is false.

Algorithm 1 provides the details of the basic DFS algorithm. We use p to save the current path information during the exploration (Line 1). We search all potential simple paths from the source vertex s with start state $A.\alpha_0$ (Line 4). If the current vertex u is target vertex t and the corresponding state α is in the set of final states (Line 7), Line 8 returns true for Reachability Query and Line 9 outputs the current path for Enumeration Query and then continues the exploration. Otherwise, Lines 10-12 loop over $N^P_{out}(u, \alpha)$ to extend path p. Line 11 checks whether we can extend p by adding

Algorithm 1: DFS

Input: The product graph $P_{G,A} = (V_P, E_P)$, source vertex s, target vertex t, the regex R and DFA A;
Output: return the answer of Reachability Query or Enumeration Query;

```
1  p ← ∅;
2  foreach v ∈ V do
3      visited[v] ← false;
4  DFS(s, A.α0, t, A.F) ;
5  Procedure DFS(u, α, t, F)
6      p ← p ∪ {u}; visited[u] ← true;
7      if α ∈ F and u = t then
8          If query is Reachability Query, print true and exit;
9          If query is Enumeration Query, print p;
10     foreach (v, β) ∈ N^P_out(u, α) do
11         if visited[v] is false then
12             DFS(v, β, t, F);
13     p ← p − {u}; visited[u] ← false;
```

v to generate p satisfying the simple path constraint. While DFS supports all expressions, it may encounter the same traps multiple times, leading to expensive time overhead.

Example 4. *In Figure 2, considering the query is $(V_0, V_{15}, (0)^* \circ (1)^*)$. We will find only one path $(V_0, V_1, V_{10}, V_{11}, V_{12}, V_{13}, V_{14}, V_{15})$ as the result. However, if the DFS explores the path following vertex ID order, the path (V_0, V_1, V_2) is found, and we cannot extend the path to the target because any path must cross V_1 twice or cannot satisfy the regular expression (i.e., V_9 cannot arrive at V_{15}). Similarly, all paths from V_1 to V_i with $2 \leq i \leq 9$ cannot reach V_{15}. However, the DFS algorithm explores these traps many times.*

3.2 Reachability Query

Bidirectional BFS (BBFS) [41] is an online algorithm for Reachability Query in this paper. BBFS detects all the potential simple paths whose labels are prefixes of expressions simultaneously from both source vertex s and target vertex t using the *breadth-first search (BFS)* strategy. When a forward path and a backward path meet at the same intermediate vertex, BBFS combines them as the whole path and checks whether the path is a simple path and matches the regex. When an eligible path is found, it means that s and t are *reachable*. Otherwise, they are *not reachable*. Because BBFS simply detects all half simple paths from both the source vertex and the target vertex, in the worst case, BBFS may explore unnecessary paths that are not in the path connecting the source and target vertices. Computing these paths uses much time and cannot get the reachability for the query. In order to address the drawback, we propose an efficient approach for solving Reachability Query with a theoretical guarantee.

3.3 Enumeration Query

Except for DFS based algorithm, a recent work by Martens and Trautner [27] shows that Enumeration Query can be solved with polynomial delay under DCE and proposes an algorithm with an $O(n^3)$ delay by extending Yen's algorithm [44]. The key idea is that for DCE, the regular shortest path is also a regular simple path. Therefore, by finding the top-k shortest paths and making k infinity, all the simple paths that match DCE can be obtained.

Algorithm 2: Framework

Input: Graph G, source and target vertices s, t, and the TRE R;
Output: returns reachability or enumeration result

1 $Pre \circ Type \circ Suf \leftarrow R$;
2 $A_{Pre} \leftarrow$ ConstructDFA(Pre) ;
3 $A_{Suf} \leftarrow$ ConstructDFA(Suf) ;
4 Algorithm 1 Line 1-3;
5 $P_f \leftarrow$ `ForwardDFS` $(s, A_{Pre}.\alpha_0, A_{Pre}.F)$;
6 $P_b \leftarrow$ `BackwardDFS` $(t, A_{Suf}.F, A_{Suf}.\alpha_0)$;
7 **foreach** $p_1 = (v_1 = s, v_2, \ldots, v_x) \in P_f$ **do**
8 **foreach** $p_2 = (u_1, u_2, \ldots, u_y = t) \in P_b$ **do**
9 $V' \leftarrow V - (\{p_1 - v_x\} \cup \{p_2 - u_1\})$;
10 $E' \leftarrow \{(u, v) | u, v \in V', (u, v) \in E\}$;
11 Query on $G' = (V', E')$ from v_x to u_1 with $Type$;
12 **Procedure** `ForwardDFS` (u, α, F)
13 Algorithm 1 Line 6;
14 $P \leftarrow \emptyset$;
15 **if** $\alpha \in F$ **then**
16 $P \leftarrow P \cup \{p\}$;
17 **foreach** $(v, \beta) \in N^P_{out}(u, \alpha)$ **do**
18 Algorithm 1 Line 11 ;
19 $P \leftarrow P \cup$ `ForwardDFS` (v, β, F) ;
20 Algorithm 1 Line 13;
21 return P;

This method is much faster than DFS. However, before finding the next shortest path, it needs to prohibit certain edges to avoid generating repeated results. This process involves checking all output paths, which may be time-consuming, especially when a large number of shortest paths have been discovered.

Our objective is to propose a more efficient algorithm than [27] to address Enumeration Query under DCE. Subsequently, we plan to extend this efficient algorithm to support all TRE.

4 ALGORITHM OVERVIEW

For TRE, solving Reachability Query and Enumeration Query under length-fixed expressions is straightforward as we only need to explore a fixed number of steps to complete the search. In this paper, we use DFS-based algorithm to compute Pre and Suf of query expressions. The most time-consuming part is dealing with $Type$, because there are Kleene stars in this part, the length of the path can be very large up to $|V|$. Therefore, finding a faster way to answer Reachability Query and Enumeration Query under downward closed expressions is crucial to designing an efficient algorithm under TRE constraints. Next, we propose the framework to compute Reachability Query and Enumeration Query.

Algorithm 2 gives a framework of our algorithm for Reachability Query and Enumeration Query. Given a graph G and a query $q(s, t, R)$, we first divide the regex R into three parts Pre, $Type$, and Suf where Pre and Suf are length-fixed expressions and $Type$ is a downward closed expression. Next, we explore the paths from the source vertex s forwardly based on Pre and record all information of suitable paths (Line 6). Similarly, we explore the paths from target vertex t backwardly based on Suf and record the path information (Line 7). Here we give the details of the forward search (Lines 13-22), while the backward search can be handled similarly.

Then, we combine the forward paths (i.e., P_f) and backward paths (i.e., P_b) in pairs and get the last vertex of the forward path (i.e., v_x) and the first vertex of the backward path (i.e., u_1) while putting other vertices of forward and backward paths into set S. Finally, we delete all vertices in set S to satisfy the simple constraint and run the efficient algorithm under $Type$ between v_x and u_1 to answer Reachability Query and Enumeration Query (Lines 8-12).

Example 5. *In figure 1(a), assume that query is* $(V_0, V_6, 0^* \circ 1^* \circ 1)$, *then we divide the regex into* $Pre = \emptyset$, $Type = 0^* \circ 1^*$ *and* $Suf = 1$ *at first. Then we do the forward and backward DFS explorations starting from* V_0 *and* V_6 *respectively. Due to the fact that* Pre *is empty, so we only do backward DFS exploration from* V_6 *and find one path* $V_5 \xrightarrow{1} V_6$. *Next, we delete vertex* V_6 *in* G *and check if there exists one simple path from* V_0 *to* V_5 *that matches* $Type$ *or find all the simple paths. Finally, we return reachable or find the simple paths* $V_0 \xrightarrow{0} V_2 \xrightarrow{0} V_3 \xrightarrow{1} V_5 \xrightarrow{1} V_6$ *and* $V_0 \xrightarrow{0} V_1 \xrightarrow{0} V_2 \xrightarrow{0} V_3 \xrightarrow{1} V_5 \xrightarrow{1} V_6$.

5 REGULAR SIMPLE PATH REACHABILITY

In this section, we introduce the algorithm named RTRE (Reachability for TRE) under $Type$. We first show an important theorem, and then present the details.

Theorem 5.1. *For DCE, if we find an arbitrary path between* s *and* t *that matches the expression, there must exist at least one simple path between these two vertices that also matches this constraint.*

Proof. If the arbitrary path does not contain any cycle, it is also a simple path that matches the regex. We consider the situation that the arbitrary path has at least one cycle. W.l.o.g., suppose that the path $p = s \xrightarrow{l_0} v_0 \cdots \xrightarrow{l_i} v_i \cdots \xrightarrow{l_j} v_j = v_i \xrightarrow{l_{j+1}} v_{j+1} \cdots \xrightarrow{l_t} t$ has one cycle, which matches the given regex R. When we delete the cycle in p, the new path will be a simple path and its corresponding label will be $l_0 \cdots l_i, l_{j+1} \cdots l_t$, which still matches the given expression based on the definition of DCE. The new path becomes the simple path between s and t that matches the regex R. If the path has more than one cycle, we can do this process repeatedly until the new path has no cycle. The final path that contains no cycle is the simple path that still matches the expression. □

According to theorem 5.1, for DCE, we can check for the existence of regular simple paths by finding a regular shortest path, which allows us to explore each edge only once.

Based on this observation, we design an algorithm based on bidirectional BFS with a block technique which also provides a theoretical guarantee for answering Reachability Query efficiently. We perform two directional BFS walks starting from the source and target vertices in the product graph $P_{G,A}$ simultaneously. For each vertex in $P_{G,A}$, we create two vectors to keep track of whether they are visited in forward and backward explorations, respectively. If we find a vertex is visited in both forward and backward searches, we return *reachable*, otherwise, we return *not reachable*. The key difference between RTRE and BBFS [41] is that RTRE explores edges in $P_{G,A}$ only once, whereas this exploration is performed multiple times in BBFS, making RTRE generally faster than BBFS. Additionally, BBFS saves all potential path information, while RTRE only records whether a vertex is explored, resulting in lower space requirements compared to BBFS.

Algorithm 3 presents the details of RTRE. Lines 1-7 complete the initialization of RTRE. Lines 9-15 record information about

Algorithm 3: RTRE

Input: The product graph $P_{G,A}$, source and target vertices s, t, and regex $Type$;
Output: returns true if t is reachable from s matching regex $Type$

```
1  Q_F, Q_B, F, B ← ∅ ;
2  A ← ConstructDFA(Type);
3  Q_F.push((s, A.α_0)) ;
4  F ← F ∪ {(s, A.α_0)} ;
5  foreach β ∈ A.F do
6      Q_B.push((t, β));
7      B ← B ∪ {(t, β)} ;
8  while Q_F ≠ ∅ and Q_B ≠ ∅ do
9      (u, α) ← Q_F.pop();
10     foreach (v, β) ∈ N^P_out(u, α) do
11         if (v, β) ∈ B then
12             return true;
13         if (v, β) ∉ F then
14             F ← F ∪ {(v, β)} ;
15             Q_F.push((v, β)) ;
16     (u, α) ← Q_B.pop();
17     foreach (v, β) ∈ N^P_in(u, α) do
18         if (v, β) ∈ F then
19             return true;
20         if (v, β) ∉ B then
21             B ← B ∪ {(v, β)} ;
22             Q_B.push((v, β)) ;
23 return false
```

forward walks, and return reachable when the forward walks meet backward walks (Lines 11-12). Lines 17-22 record information about backward walks and the remaining steps are similar to the process of forward walks. If we finish exploring forward walks or backward walks (Line 8), we return unreachable.

Example 6. *Assume that the query is* $(V_0, V_5, 0^* \circ 1^*)$ *in Figure 1(a). We need to find a simple path from* $(V_0, 0)$ *to* $(V_5, 0)$ *or* $(V_5, 1)$ *in* $P_{G,A}$. *Table 3 shows details of forward and backward explorations and records vertices that have been explored in each step. Step 0 means initialization. When we perform the forward search in step 3, we find the vertex* $(V_3, 0)$ *has been explored in both forward and backward searches. Hence, we can stop the search and return reachable.*

Theorem 5.2. *The time complexity of RTRE is* $O(n \cdot k + m \cdot k^2)$, *and the space complexity is* $O(n \cdot k + m)$, *where* n *is the number of vertices in* G, m *is the number of edges in* G *and* k *is the number of states in the corresponding DFA* A *of given regex.*

Proof. If there are k states in DFA A, then there are at most $n \cdot k$ vertices and $m \cdot k^2$ edges in the corresponding product graph $P_{G,A}$. Since every edge in $P_{G,A}$ is explored only once, the time complexity is $O(n \cdot k + m \cdot k^2)$. Each vertex with every state may be saved. The space complexity is $O(n \cdot k + m)$. □

6 REGULAR SIMPLE PATH ENUMERATION

Compared with the Pre and Suf in a query, the challenge of regular simple path enumeration is still the $Type$. Hence we propose some pruning techniques to help us accelerate enumeration. Firstly, we propose a candidate detection method for pruning unnecessary vertices. Secondly, we try to prune the search branch that does not contain any eligible path by previous searching. Thirdly, we improve the pruning rules by the property of DCE and propose a faster and polynomial delay enumeration algorithm.

Table 3: The exploration of RTRE

Step	Forward search	Backward search
0	$(V_0, 0)$	$(V_5, 0), (V_5, 1)$
1	$(V_0, 0), (V_1, 0), (V_2, 0)$	$(V_5, 0), (V_5, 1)$
2	$(V_0, 0), (V_1, 0), (V_2, 0), (V_7, 1)$	$(V_5, 0), (V_5, 1), (V_3, 0)$
3	$(V_0, 0), (V_1, 0), (V_2, 0), (V_7, 1), (V_3, 0)$	

6.1 Candidate Detection

When the source and target vertices s and t are given, it is possible to identify many vertices in the graph that do not need to be explored because they cannot reach t under the expression's constraint alone (i.e., ignoring the simple constraint temporarily). In this subsection, we present a technique called *Candidate Detection*, which detects all the candidate vertices in the product graph $P_{G,A}$.

We determine the candidate vertices by using the backward BFS strategy from the target vertex t with the final states of DFA A. Algorithm 4 provides the details. During the exploration from t with the final states (Lines 6-11), we record every vertex in $P_{G,A}$ that has been visited (Line 10). All these visited vertices are the candidate vertices. We will delete other vertices along with their corresponding edges, to obtain a new product graph P' (Line 12).

6.2 Pruning by Conflict Sets

After obtaining the new product graph P', we aim to identify exact simple paths that satisfy the given regex. While DFS is a straightforward approach for enumerating all simple paths, it can become inefficient due to exploring futile branches in the search tree. To address this, we seek to identify and characterize these futile branches, known as conflict sets. By employing a DFS strategy, we traverse the search tree, periodically encountering vertices such as M. If M cannot reach the target vertices, it implies the existence of conflict vertices (ones explored repeatedly in paths from the source vertex to M and the subtree rooted at M). To streamline future searches, we leverage conflict information obtained during subtree exploration to prune redundant searches.

Computing Conflict Sets. Next, we describe how to compute the conflict set C_M of node M in a bottom-up fashion. Initially, we define the leaves of the search tree, which can be categorized into the following three types, w.l.o.g., assuming that we have explored the simple path p_c and arrived at node M in $P_{G,A}$.

(1) A leaf is a *conflict node* in the subtree rooted at M if and only if we should stop exploring this node due to the simple constraint.
(2) A leaf can be a target node, indicating we find a result.
(3) Otherwise, we refer to this leaf as a *normal node*.

Now, we compute the conflict set of node M by using its children and their corresponding conflict sets. Assume that node M has k children $M_1, \cdots, M_k$, and we have already computed the conflict set $C_{M_1}, \cdots, C_{M_k}$ of these children, respectively. Based on the following cases, we can compute the conflict set C_M of node M. If there exists a child node M_i such that M_i is the target node, we set $C_M = \emptyset$. Otherwise, if child nodes are all normal nodes, we set

Algorithm 4: CandidateDetection

Input: The product graph $P_{G,A}$, target vertex t, DFA A;
Output: The pruned product graph;

$Q \leftarrow \emptyset$;
$B \leftarrow \emptyset$;
foreach $\alpha \in A.F$ **do**
  Q.push((t, α));
  $B \leftarrow B \cup \{(t, \alpha)\}$;
while $Q \neq \emptyset$ **do**
  $(u, \alpha) \leftarrow Q.pop()$;
  foreach $(v, \beta) \in N_{in}^{P}(u, \alpha)$ **do**
    if $(v, \beta) \notin B$ **then**
      $B \leftarrow B \cup (v, \beta)$;
      $Q.push((v, \beta))$;
return $P' = (V' = \{(u, \alpha) | B[u][\alpha] = true\}, E' = \{(u, v) | u, v \in V'\})$

Algorithm 5: Conflict DFS

Input: Product graph $P_{G,A}$, source and target vertices s, t, and regex $Type$;
Output: all simple paths between s and t matching $Type$

$A \leftarrow$ ConstructDFA($Type$);
$P' \leftarrow$ CandidateDetection($P_{G,A}$,t, A);
Initialize Conflict set C to empty;
Return `Conflict DFS`$(s, A.\alpha_0, t, A, \{s\})$;
Procedure `Conflict DFS`(u, α, t, A, p)
  Result $\leftarrow \emptyset$;
  if $u = t$ *and* $\alpha \in A.F$ **then**
    return $\{p\}$;
  foreach $(v, \beta) \in N_{out}^{P'}(u, \alpha)$ **do**
    if $p \cap C[v][\beta] \neq \emptyset$ **then**
      break;
    if $v \notin p$ **then**
      Result $\leftarrow$ Result $\cup$ `Conflict DFS`$(v, \beta, t, A, p \cup \{v\})$;
  Update the conflict set for (u, α);
  return *Result*;

$C_M = \cap_{i=1}^{k} C_{M_i}$. Otherwise, we use set C to record all the *conflict nodes*. If there are at least two distinct nodes in C, we set $C_M = \emptyset$. Otherwise, we set $C_M = (\cap_{M_i \in C} C_{M_i}) \cap C$.

Note that the conflict sets may be computed multiple times for the same node, and the updates are done in an additional manner rather than a coverage manner, which means that new values are inserted into the conflict sets without deleting old values. Additionally, in this part, we only consider the simple constraint based on node information, so in the conflict sets, we only include the node information while ignoring the state information. For example, if a node (u, α) has two conflict nodes (v, β_1) and (v, β_2), even though these two nodes have different states, the conflict set of (u, α) is v but not $\emptyset$ because the node v is the essential reason why exploration is stopped, rather than the state β_1, β_2.

Algorithm 5 shows how to utilize conflict sets for pruning the search tree. Line 10 illustrates our pruning technique. Specifically, if we find one node in the current path that is present in its conflict set, we can terminate the exploration, as the new path after exploration must conflict with the simple constraint. Note that Conflict DFS can handle any expressions, but in some bad cases, it may still cost exponential steps to return results.

6.3 Pruning by Block

If the current partial path does not include the vertices in the conflict set, we cannot determine whether the current search branch contains an eligible simple path. To address the limitation of conflict sets, we propose a more efficient method called ETRE (Enumeration for TRE) than Conflict DFS under $Type$.

After candidate detection, this method involves using a blocked set B to determine whether each vertex in the new product graph P' needs to be explored. Specifically, in the DFS exploration, we will put some vertices into the blocked set after they have been visited. Note that the vertex in the blocked set cannot be explored until it is not in the blocked set. We will explore the vertex (u, α) only if it is not in B and the current path p does not contain u. Otherwise, we terminate the exploration and backtrack. If the blocked set encompasses a vertex, we refer to it as a blocked vertex (or simply, we block this vertex).

Considering the query $(V_0, V_7, 0^* \circ 1^*)$ in Figure 1(a). We show how the blocked set evolves and how repetitions of useless explorations can be pruned. At this stage, readers do not need to worry about how to update the blocked set when backtracking to find the next regular simple path, as this process will be addressed after this example. In Figure 1(a), assume that we explore the path $V_0 \xrightarrow{0} V_1 \xrightarrow{0} V_2 \xrightarrow{0} V_3 \xrightarrow{1} V_4$ first. The corresponding exploration in the product graph is as follows:

- $(V_0, 0) \rightarrow (V_1, 0)$**:** Put $(V_1, 0)$ into B and add V_0, V_1 to p.
- $(V_1, 0) \rightarrow (V_2, 0)$**:** Put $(V_2, 0)$ into B and add V_2 to p.
- $(V_2, 0) \rightarrow (V_3, 0)$**:** Put $(V_3, 0)$ into B and add V_3 to p.
- $(V_3, 0) \rightarrow (V_4, 1)$**:** Put $(V_4, 1)$ into B and add V_4 to p.
- Then, we do not explore vertex $(V_1, 1)$ because V_1 is in the current path p. We backtrack to $(V_3, 0)$ to find other paths. So we reach $(V_5, 1)$ and add $(V_5, 1)$ to B. Then, we attempt to explore the vertex $(V_4, 1)$ again. However, due to $(V_4, 1)$ being in B, we abort this search and backtrack.
- Next, we will backtrack to vertex $(V_1, 0)$ and find one result.

While this appears promising, we must acknowledge a major issue that has been overlooked. During backtracking to find the next path, previously blocked vertices may become available again, which can impact the accuracy of the blocked set. We extend the previous example to illustrate this problem. After finding the result $V_0 \xrightarrow{0} V_1 \xrightarrow{1} V_7$, we will explore vertex $(V_2, 0)$ as we have finished all the exploration rooted at $(V_1, 0)$. However, we cannot explore vertex $(V_2, 0)$ since $(V_2, 0)$ is still in B. Nevertheless, there exists a path $V_0 \xrightarrow{0} V_2 \xrightarrow{0} V_1 \xrightarrow{1} V_7$ that satisfies our requirements.

The problem is that the blocked set is not updated in a timely manner. Therefore, we need to evaluate under which conditions the vertex could be unblocked (i.e., delete it from the blocked set) at the same time when we block it. There are two cases in which we should update the blocked set. Firstly, when we find a path that satisfies our request, we should unblock all the vertices in this path. Secondly, there are cases where we need to unblock certain vertices while unblocking others. For example, if we unblock vertex $(V_1, 0)$, we should also unblock vertex $(V_3, 0)$, as if $(V_1, 0)$ could be explored again, $(V_3, 0)$ will be also available.

Therefore, we add an unblock list for every vertex, which maintains a list of vertices that should be unblocked when the vertex itself is unblocked. This allows for cascading unblock operations.

Algorithm 6: Unblock

```
Input: Block set B, unblock list U, vertex u, state α
Procedure Unblock(B, U, u, α)
    B ← B − {(u, α)} ;
    foreach (v, β) ∈ U[u][α] do
        Unblock(B, U, v, β) ;
    U[u][α] ← ∅
```

Algorithm 7: ETRE

```
Input: product graph P_{G,A}, source and target vertices s, t, and Regex Type;
Output: all simple paths between s and t matching Type
1  A ← ConstructDFA(Type);
2  P' ← CandidateDetection(P_{G,A}, t, A);
3  p ← {s};
4  B ← ∅;
5  foreach (u, α) ∈ P' do
6      U[u][α] ← ∅;
7  return ETRE (s, A.α_0, t, p, P');
8  Procedure ETRE (u, α, t, p, P')
9      B ← B ∪ {(u, α)} ;
10     Result ← ∅ ;
11     foreach (v, β) ∈ N^{P'}_{out}(u, α) do
12         if v = t and β ∈ A.F then
13             Result ← Result ∪ {p};
14         if v ∉ p and (v, β) ∉ B then
15             Result ← Result ∪ ETRE (v, β, t, p ∪ {v}, P')
16     if Result ≠ ∅ then
17         Unblock(B, U, u, α);
18     else
19         foreach (v, β) ∈ N^{P'}_{out}(u, α) do
20             U[v][β] ← {(u, α)};
21     return Result;
```

In our example, when we find the result $V_0 \xrightarrow{0} V_1 \xrightarrow{1} V_7$, the vertex $(V_1, 0)$ will be unblocked. We delete $(V_1, 0)$ from the blocked set. Since $(V_3, 0)$ is in the unblock list of $(V_1, 0)$, it will be deleted as well, leading to the deletion of $(V_2, 0)$, and this process continues until the unblock list is empty. This ensures that all the vertices that need to be unblocked are properly updated, allowing for correct handling of cascade unblock operations.

Overall, ETRE could be considered as an improved method of conflict sets. Conflict sets record local information, which can lead to repeated exploration of unproductive paths when previous paths change. On the other hand, ETRE maintains global information to efficiently prune the search space, ensuring that no futile searches are explored between two results outputs. The pseudocode for the ETRE algorithm is presented in Algorithm 7.

Specifically, we block every vertex during the exploration (Line 9). When we meet the blocked vertex or the current path contains its vertex information, we will terminate the further exploration (Line 14). When we finish the exploration of one vertex and find one result, we will unblock this vertex (Lines 16-17). Note that the unblock process is recursive until no vertex can be unblocked. Otherwise, we will keep blocking this vertex and record the information about how can we unblock it (Lines 19-20).

Theorem 6.1. *ETRE returns all the simple paths between the start vertex s and the target vertex t that match the given DCE R.*

Proof. First of all, it is important to realize that ETRE is a truncated depth-first search: all paths are explored except for paths that contain blocked vertices. There are two cases where a vertex can be blocked. In the first case, the vertices in the currently explored path are blocked. When we arrive at them again in the exploration, we need to terminate the exploration since it leads to a non-simple path. In the second case, during backtracking, even if some vertices are not in the current path, they are still blocked. Now assuming that we have explored the path $s \rightarrow \cdots \rightarrow v_i \rightarrow \cdots \rightarrow v_k$, if we have already explored the subtree rooted at v_k and determined v_k should not be unblocked, it implies that v_k is not in a result path. This indicates that v_k must conflict with a vertex in a later part of the path, represented by v_i. Therefore, v_k will be unblocked only if v_i is unblocked. However, v_i can only be unblocked if we have finished exploring the subtree rooted at v_i, which means it does not affect the subsequent exploration. Considering the recursive nature of the process, block and unblock techniques can not affect the correctness of the exploration. □

Theorem 6.2. *The total time complexity of ETRE is $O((c+1)(n \cdot k + m \cdot k^2))$, and the space complexity is also $O(n \cdot k + m \cdot k^2)$, where c is the number of results.*

Proof. The proof of this theorem is based on the design that the only way to unblock one vertex is by a call Unblock, which only happens when a path is output. Whenever a path $s \rightarrow v_1 \cdots \rightarrow v_i \rightarrow t$ is output, Unblock will be executed for v_i (i.e., the prefix path is $s \rightarrow v_1 \cdots \rightarrow v_i$), then for v_{i-1}, until it is called for v_1. However, every call for Unblock will unblock different vertices. Indeed, if now the prefix path is $s \rightarrow v_1 \cdots \rightarrow v_j$, a call to Unblock only unblocks the vertices that at this moment cannot be used in a path to the target vertex t, but once v_j is unblocked, they will become available again. Therefore, every edge can be explored at most twice between two paths output. Hence, either a path will be output after $O(m \cdot k^2)$, or all the vertices will be blocked and we terminate the algorithm. The term $O(n \cdot k)$ is for the initialization of the blocked switches of all vertices at the start of the algorithm. We use $O(n \cdot k)$ space to block vertices and $O(m \cdot k^2)$ space to record the unblock list U. □

7 REACHABILITY VERTEX PAIR QUERY

The problem of returning the pairs of vertices connected by simple paths satisfying a given regular expression is NP-hard. [28] proposes an efficient method that can address this problem in polynomial time under restricted regular expressions (which are equal to downward closed expressions, the details can be seen in [28]) even considering the simple paths. Unfortunately, using the method in [28] to evaluate queries under the expression form $Pre \circ Type$ takes exponential time. Note that $Type \circ Suf$ is very similar to $Pre \circ Type$. Hence, we only discuss $Pre \circ Type$.

Algorithm 8 gives the details of our method. To be specific, the expression is firstly broken into two parts (i.e., Pre and $Type$). Then we use the DFS method to find all the vertices that the start vertex s can reach under Pre constraint through simple paths and record the corresponding simple paths (Line 4). Then for every path, we delete all the vertices in the path except the final vertex (Line 6) and run the efficient algorithm under $Type$ from v_s to answer the

Algorithm 8: Reachability Vertices Pair Query

Input: Graph $G = (V, E, \mathcal{L}, \phi)$ and the expression R;
Output: returns the pairs of nodes connected by simple paths satisfying R

1 $Pre \circ Type \leftarrow R$; $Q, Result \leftarrow \emptyset$;
2 $A_{Pre} \leftarrow$ ConstructDFA(Pre); $A_{Type} \leftarrow$ ConstructDFA($Type$) ;
3 **foreach** $s \in V$ **do**
4 $P_f \leftarrow$ ForwardDFS $(s, A_{Pre}.\alpha_0, A_{Pre}.F)$;
5 **foreach** $p_1 = (v_1 = s, v_2, \ldots, v_x) \in P_f$ **do**
6 $V' \leftarrow V - (\{p_1 - v_x\})$; $E' \leftarrow \{(u, v) | u, v \in V', (u, v) \in E\}$;
7 $Q.push((v_x, A_{Type}.\alpha_0))$; $F \leftarrow \emptyset$;
8 **while** $Q \neq \emptyset$ **do**
9 $(u, \alpha) \leftarrow Q.pop()$;
10 **foreach** $(v, \beta) \in N^P_{out}(u, \alpha)$ **do**
11 **if** $\beta \in A_{Type}.F$ *and* $(s, v) \notin Result$ **then**
12 $Result \leftarrow Result \cup (s, v)$;
13 **if** $(v, \beta) \notin F$ **then**
14 $F \leftarrow F \cup \{(v, \beta)\}$;
15 $Q_F.push((v, \beta))$;
16 **return** *Result*

query (Lines 8-15). Specifically, we can find all the vertices that one vertex can reach by finding single source shortest paths. We design an algorithm based on BFS with a block technique to find the vertices. We perform a BFS walk from the source vertice in the product graph $P_{G,A}$ and create a vector to keep track of whether the vertices in $P_{G,A}$ are visited in the exploration, which contributes to the exploration from every vertex can be only constructed once (Lines 13-15). During the exploration, we record all the visited vertex with the final states as results (Lines 11-12). We do the above process for all the vertices in G and then we find all pairs of vertices connected by simple paths satisfying the expression (Line 3).

Theorem 7.1. *The time complexity of Algorithm 8 is* $O(n \cdot (m_d)^{k_p} + c(n \cdot k_t + m \cdot k_t^2))$, *and the space complexity is* $O((m_d)^{k_p} + n \cdot k_t + m)$, *where* m_d *is the maximal degree of* G, c *is the number of all forward paths, and* k_p *and* k_t *is the number of states in the corresponding DFA of* Pre *and* $Type$, *respectively.*

Proof. It takes $O(n \cdot (m_d)^{k_p})$ time to find all the forward paths, and we need $O(n \cdot k_t + m \cdot k_t^2)$ to find the results for each forward path. It takes at most $O((m_d)^{k_p})$ space to save the forward paths, and we need $O(n \cdot k_t)$ space to record the information about whether vertices are visited. The final time cost is $O(n \cdot (m_d)^{k_p} + c(n \cdot k_t + m \cdot k_t^2))$, and the final space cost is $O((m_d)^{k_p} + n \cdot k_t + m)$. □

8 EXPERIMENTS

Datasets: Table 4 lists the basic information of 17 real-graphs used in our experiments, most of which are used in the related work [32, 37]. Eight of the graphs have natural edge labels, while for the remaining graphs without edge labels, we synthetically generate labels that are exponentially distributed with $\lambda = \frac{|\mathcal{L}|}{\alpha}$, where $\alpha = 1.7$. For all undirected graphs (i.e., HS, BG, FC and FS), an undirected edge was replaced by two directed edges.

Comparisons. We investigate the following methods in Reachability Query and obtain source codes from the respective authors.

- **BBFS [41]:** the baseline algorithm.
- **ARRIVAL [41]:** the state-of-the-art approximate method.

Table 4: Statistics of datasets used in the experiments. ($K = 10^3, M = 10^6, B = 10^9$)

Name	Dataset	$\|V\|$	$\|E\|$	$\|\mathcal{L}\|$	$\frac{\|E\|}{\|V\|\|\mathcal{L}\|}$	Type	Syn
AD	Advogato	6.5K	51.1K	4	1.96	Trust	
EC	econ-psmigr1	3.1K	543K	8	21.89	Economic	✓
TR	Wiki-trust	139K	740K	8	0.66	Interaction	✓
HS	StringsHS	19K	1.24M	8	8.15	Biological	
BG	BioGrid	64K	1.5M	7	3.34	Biological	
FC	StringsFC	19K	2.04M	8	13.42	Biological	
ND	NotreDame	326K	1.47M	8	0.56	Web	✓
SF	Web-stanford	282K	2.3M	8	1.01	Web	✓
BK	Baidu-baike	416K	3M	8	0.90	Web	✓
GG	Web-google	876K	5M	8	0.71	Web	✓
DA	Rec-dating	169K	17M	10	10.05	Recommendation	
YT	Youtube	14.9K	13.6M	5	182.55	Social	
EP	Soc-Epinsion1	75K	508K	8	0.84	Social	✓
SO	StackOverflow	2.6M	63M	3	8.07	Social	
ZS	zhishihudong	2.4M	18.9M	8	0.98	Miscellaneous	✓
FS	friendster	65M	2.6B	30	1.33	Miscellaneous	✓
WD	Wikidata	296M	958M	5419	0.001	Miscellaneous	

- **RTRE:** Algorithm 2 + Algorithm 3.

The methods studied in Enumeration Query are the following:

- **Yen's algorithm [27]:** the version of Yen's algorithm [44] for Enumeration Query under DCE.
- **DFS:** the baseline algorithm.
- **Conflict DFS:** Algorithm 5.
- **ETRE:** Algorithm 2 + Algorithm 7.

We investigate two methods in Reachability Vertices Pair Query:

- **Wood's algorithm [28]:** the start-of-the-art method.
- **RVPM :** Algorithm 8.

All algorithms are implemented in C++ and compiled with g++ with O3 optimization. The experiments are performed on a machine with an Intel Xeon 2.1GHz CPU and 256G memory.

Queries. We choose the top-5 frequent queries from Table 1 [2]. Tabel 5 lists the expressions we use in experiments. The number k in Q_4 and Q_5 is chosen from 2 to 6. Q_6 must consist of $Type$, and Pre and Suf are randomly added to Q_6. The length of labels in $Type$ is chosen from a range of 2 to 6, and the number in Pre and Suf is from a range of 1 to 4. We make two different forms for $Type$, i.e., $R = (l_1 + l_2 + \cdots + l_k)^*$ and $R = (l_0)^* \circ (l_1)^* \circ \cdots \circ (l_k)^*$. The form of Pre and Suf is $R = (l_0) \circ (l_1) \circ \cdots \circ (l_k)$.

The source vertex, target vertex, and label are chosen randomly from the graph. We generate 1000 queries for Reachability Query comprising 500 true queries and 500 false queries. Additionally, we have created 100 queries for Enumeration Query, ensuring the existence of at least one result. Due to limited space, we cannot include all experimental results in this paper. The additional experimental results and discussions are given in the long version of our paper[3].

8.1 Performance on Reachability Query

8.1.1 Comparison with BBFS [41]. We set a time limit as 50 seconds for BBFS and compare the performance of BBFS with RTRE. Tabel 6, 7, 9 demonstrate that RTRE is up to three orders of magnitude faster than BBFS on almost of graph datasets under recursive queries (i.e.,

[2] We do not choose A since the final result path only has one edge for this query.
[3] https://github.com/Newth-QiLiang/Regular-Simple-Path-Queries.

Table 5: Queries used in experiments

Name	Type	Name	Type
Q_1	a^*	Q_4	$(a_1 + a_2 \cdots + a_k)^*$
Q_2	$a \circ b^*$	Q_5	$a_1 \circ a_2 \cdots \circ a_k$
Q_3	$a \circ b^* \circ c^*$	Q_6	Random TREs

Q_1, Q_2, Q_6) [4]. However, because RTRE makes no effect on non-recursive queries, RTRE shows the same efficiency as BBFS.

Table 6: Query time in microseconds for Q_1. (TQ: true query, FQ: false query, Rec: recall, OOM: out of memory, OOT: initialization time > 12h.)

Name	BBFS		RTRE		ARRIVAL		
	TQ	FQ	TQ	FQ	TQ	FQ	Rec
AD	217	53	**53**	**21**	112	22	0.88
BG	870K	311K	891	**214**	**513**	297	0.87
BK	670K	47K	**532**	**33**	634	213	0.76
DA	2.4K	149	**648**	**53**	2.6K	971	0.93
EC	131	52	**33**	**15**	139	62	0.99
SO	11K	265	1K	46	2.5K	552	0.98
TR	375	46	**120**	**29**	343	32	0.97
YT	292K	7.7M	**508**	11K	934	**632**	0.63
WD	5.2M	33K	**443K**	**36**	OOT	OOT	OOT
FS	OOM	OOM	**7.2K**	**46**	OOT	OOT	OOT

Table 7: Query time in microseconds for Q_2.

Name	BBFS		RTRE		ARRIVAL		
	TQ	FQ	TQ	FQ	TQ	FQ	Rec
AD	188	49	**29**	**4**	127	40	0.7
BG	97K	127K	**254**	**41**	408	206	0.83
BK	360K	30K	**378**	**20**	629	751	0.64
DA	6.1K	591	**655**	**32**	2.6K	1.7K	0.82
EC	171	165	**28**	**27**	222	155	0.99
SO	201K	4.4K	**1.3K**	**75**	5.2K	676	0.96
TR	920	51	**174**	**5**	488	96	0.95
YT	7.7K	6.2M	**234**	7.6K	786	**474**	0.41
WD	4.6M	36K	**325K**	**75**	OOT	OOT	OOT
FS	OOM	OOM	**7.1K**	**22**	OOT	OOT	OOT

8.1.2 Comparison with ARRIVAL [41]. ARRIVAL approximates the diameter of graphs to initialize the parameter *walkLength*, which may be time-consuming. In our comparison of ARRIVAL and RTRE under recursive queries, we observe that RTRE demonstrates comparable efficiency to ARRIVAL since ARRIVAL limits the path length and number to reduce the exploration, and RTRE optimizes the exploration under DCE constraint. ARRIVAL, on the other hand, may return approximate results with low *recall* (e.g., BG with 34% on Q_6 and YT with 39% on Q_6). The discrepancy in the *recall* can be attributed to the fact that *recall* is based on strongly connected graphs, whereas real-world graphs are typically not strongly connected.

[4]Due to limited space, we only show the results on 10 datasets and omit the results under Q_3 and Q_4, they can be seen in https://github.com/Newth-QiLiang/Regular-Simple-Path-Queries.

Table 8: Query time in microseconds for Q_5.

Name	BBFS		RTRE		ARRIVAL		
	TQ	FQ	TQ	FQ	TQ	FQ	Rec
AD	59	62	59	62	**43**	**44**	0.7
BG	**84**	917	**84**	918	430	**147**	0.002
BK	242	13.9K	243	13.9K	**228**	**107**	0.95
DA	2.6K	121K	2.6K	121K	**1.3K**	**1K**	0.45
EC	2.6K	154K	2.6K	154K	**273**	**341**	0.9
SO	**704**	149K	706	149K	965	**1.5K**	0.96
TR	**394**	2.5K	395	2.5K	460	**97**	0.71
YT	346	291K	348	291K	**292**	**223**	0.24
WD	**2.4K**	**47**	2.4K	50	OOT	OOT	OOT
FS	156K	144K	156K	144K	OOT	OOT	OOT

Table 9: Query time in microseconds for Q_6.

Name	BBFS		RTRE		ARRIVAL		
	TQ	FQ	TQ	FQ	TQ	FQ	Rec
AD	11.6K	1.5M	**104**	127	212	**64**	0.64
BG	4.3M	7.5M	5.2K	15.6K	**1.2K**	**618**	0.34
BK	13.1M	1.1M	**9.4K**	14.9K	16.2K	**2.1K**	0.31
DA	133K	1.3K	**2.6K**	**473**	12.1K	3.6K	0.66
EC	803	344	**166**	134	318	**85**	0.99
SO	364K	3.1M	5.2K	265K	**4.8K**	**1.1K**	0.91
TR	46K	170K	**1.0K**	608	1.4K	**43**	0.71
YT	66.2K	24.9M	**813**	17.3K	1.9K	**1.5K**	0.39
WD	51.9K	167.5	**1.3K**	**165**	OOT	OOT	OOT
FS	OOM	OOM	**213K**	**3.9M**	OOT	OOT	OOT

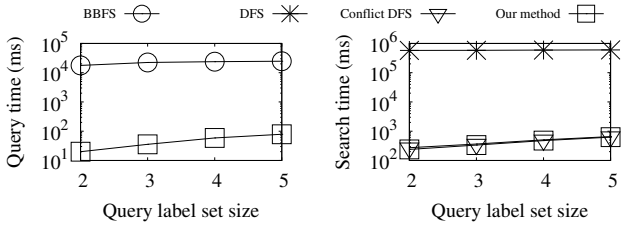


Figure 3: Impact of the number of labels in the query regex.

8.1.3 Performance against Query-Label Size. We conduct an analysis of query time for RTRE and BBFS with respect to the number of labels in the *Type* on YT, as shown in the left of Figure 3. We observe that the query time increases linearly as the number of labels increases since the number of paths that match the regex also increases with the increase in the number of labels, requiring more exploration of paths. Nevertheless, RTRE is still at least 2 orders of magnitude faster than BBFS on YT.

8.2 Performance on Enumeration Query

For each algorithm, we evaluate their performance based on *search time*, defined as the duration from the start of a query until the first 1000 results are obtained. We set 10 minutes as the time limit.

8.2.1 Comparison with Yen's algorithm [27]. Recall that Yen's algorithm only supports DCE, so we compare the performance of

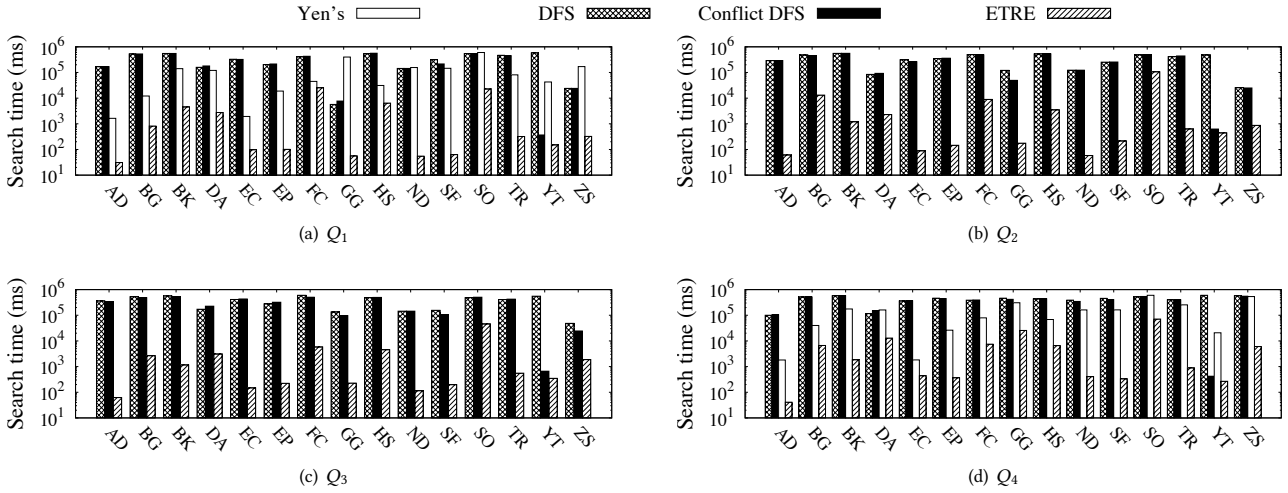


Figure 4: Search time comparison on Enumeration Query.

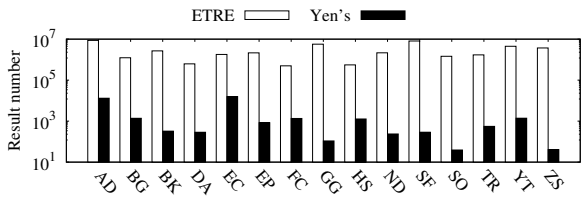


Figure 5: The result number comparison.

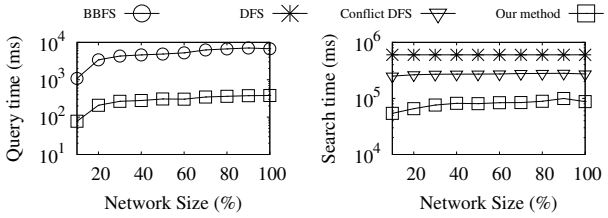


Figure 6: Time comparison on WD having different sizes.

ETRE and Yen's algorithm specifically on Q_1 and Q_4. Figure 4(a) and 4(d) illustrates the average search time on Q_1 and Q_4, respectively. ETRE is nearly 2-3 orders of magnitude faster on almost of datasets compared to Yen's algorithm. We also use another metric named result number, which means the number of simple paths that can be found within 1 minute, to test the performance of Yen's algorithm and ETRE. We randomly generate DCE and compare the performance of ETRE and Yen's algorithm. Figure 5 shows that ETRE can find over 100 times more paths than Yen's algorithm.

The superior performance of ETRE can be attributed to a key factor. Yen's algorithm algorithm requires deleting some edges in the graph to avoid repeated results, which incurs a rapidly growing cost as the number of results increases. In contrast, ETRE only needs to unblock some vertices in the graph, resulting in nearly constant cost, which contributes to superior performance.

8.2.2 Comparison with DFS and Conflict DFS. Figure 4 shows the average search time of DFS, Conflict DFS and ETRE [5]. In comparison, ETRE is nearly 1000 times faster than DFS and Conflict DFS on most graphs, as they tend to repeatedly explore unnecessary paths while ETRE avoids these useless explorations as much as possible.

8.2.3 Performance against Query-Label Size. We illustrate the impact of the number of labels in the *Type* on the search time of ETRE in the right of Figure 3. The search time increases linearly with the number of labels, indicating that ETRE is less sensitive to the number of labels in the query regex.

[5]Due to limited space, we omit the results under Q_5 and Q_6, they can be seen in the long vision of our paper.

8.3 Scalability Evaluation

Figure 6 illustrates that query time for Reachability Query and search time for Enumeration Query generally increase with network size for most algorithms except between 90% and 100% size. The search space grows up varying graph size increasing. However, we just have to find limited eligible paths (1 path for the reachability problem and 1000 paths for the enumeration problem). More edges may result in more eligible paths in the search space. DFS and Conflict DFS have little increase since they cannot finish most of the queries within the time limit. Overview, the query time of our methods increases linearly and our methods (i.e., RTRE and ETRE) outperform the baselines (i.e., BBFS and DFS).

8.4 Memory Evaluation

We show the extra memory requirements for Q_6. We do not compute the graph memory cost since all methods must save the graph.

For Reachability Query, Figure 7 demonstrates that BBFS costs the most extra memory since it needs to contain all the potential simple paths that require lots of memory. In general, RTRE requires similar memory to ARRIVAL since RTRE and ARRIVAL use two different methods to reduce the memory cost. For Enumeration Query, Figure 8 illustrates that DFS needs the least extra space as

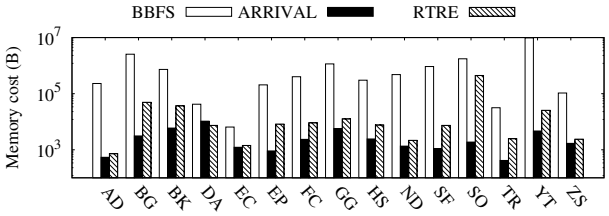


Figure 7: Memory comparison on Reachability Query.

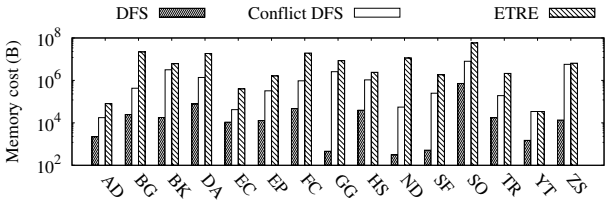


Figure 8: Memory comparison on Enumeration Query.

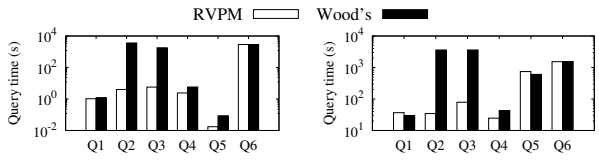


Figure 9: Query time comparison.

it only records the current exploring path which is related to the depth of simple paths. Conflict DFS needs more memory since they need BFS exploration and record extra information (such as the information about candidate vertices). ETRE costs the most extra memory because it does not only record the candidate vertices but also records the information about when the block vertices can be unblocked. Please note the unit of memory cost is Byte, so even if ETRE costs most space, its cost is less than 100 MB, which is a reasonable space cost.

8.5 Reachability Vertices Pair Query Evaluation

We set the time limit as 1 hour and compare the performance of our method with Wood's algorithm on two datasets (i.e., AD and EC). Figure 9 demonstrates that our method has comparable efficiency to Wood's algorithm on Q_1, Q_4, and Q_5, and both of them can be completed within 1 hour. However, our method is up to three orders of magnitude faster than Wood's algorithm on Q_2 and Q_3. It is because although these two types are very simple, Wood's algorithm takes exponential time to evaluate the queries while our method can finish these queries in polynomial time. Unfortunately, both of these two algorithms are out of time in Q_6 since it will take exponential time to evaluate the random queries under TREs.

9 RELATED WORK

Regular Path Query (RPQ): Most of the existing works focus on Reachability Vertices Pair Query. Koschmieder and Leser [21] use the rare label to improve efficiency. Wang et al. [43] answer regular path queries by evaluating partial answers evaluation. Jachiet et al. [18] propose a variation of the relational algebra to solve this problem. Other methods [14, 22, 25, 42] for RPQ evaluation construct indexes to optimize RPQ evaluation. Na et al. [30] propose a lightweight transitive closure to evaluate RPQs. Arroyuelo et al. [4] evaluate RPQs with high space efficiency. Pacaci et al. [31] focus on evaluating RPQs on streaming graphs. We do not compare these works since they do not consider simple path semantics.

Simple Path Enumeration: Birmele et al. [6] investigate the s-t path enumeration problem in undirected graphs, which cannot extend to directed graphs. Tarjan [38] and Johnson [20] propose algorithms for enumerating simple cycles in directed graphs. A recent work [23] proposes an efficient algorithm for enumerating all simple temporal cycles in temporal graphs. These methods cannot extend straightly to solve RSPQs since label information should be considered. Recent researches focus on hop-constrained s-t simple path enumeration. Several theoretical works [16, 36] achieve polynomial delay. Peng et al. [33] propose BC-DFS and JOIN, achieving high efficiency. Sun et al. [37] also propose an index-based method for real-time enumeration of hop-constrained s-t simple paths. HP-index [35] maintains paths between hot vertices and enables real-time detection of hop-constrained cycles in large dynamic graphs. These algorithms are not suitable for solving the RSPQs as they are based on the length constraint whereas almost of RSPQs do not limit the length of paths. The purpose of the above methods is similar to ours. We all want to achieve polynomial delay and avoid unnecessary paths as much as possible. However, our method is based on an efficient pruning technique, which contributes to exploring the useless paths only once, and it considers label constraints as well as avoids length limitations.

10 CONCLUSION

In this paper, in order to address two problems of RSPQs efficiently, we summarize a type of regular expression that covers over 99% of real-world queries. Then, we propose an efficient algorithm framework to solve both Reachability Query and Enumeration Query. Our experimental analyses on extensive datasets demonstrate that our methods have comparable efficiency to the approximate method and outperforms significantly the state-of-the-art exact methods.

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